Elementary Geometry for Teachers

Homework adaptation for the Standards Edition

This booklet contains homework for *Elementary Geometry for Teachers* (EGT) for use with the Standards Edition of the *Primary Mathematics* textbooks

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Adapted to the *Primary Mathematics* Standards Edition by Benjamin Ellison and Daniel McGinn.

How to use these notes

The textbook *Elementary Geometry for Teachers* is designed to be used in conjunction with the Primary Mathematics (U.S. Edition) textbooks, and many of the homework exercises refer to specific pages in these books. After the 2008 publication of the Standards Edition of the Primary Mathematics books some readers, most notably in-service teachers in California, have had ready access to the Standards Edition, but not to the U.S. Edition books. This reference booklet delineates the changes to the text and the homework assignments needed for readers who prefer to use the Standards Edition instead of the U.S. Edition.

You will need *Elementary Geometry for Teachers* (EGT) and the following Primary Mathematics textbooks:

- Primary Math Standards Edition 2B (needed for only 3 homework exercises).

- Primary Math Standards Edition 3B.

- Primary Math Standards Edition 4A and 4B.

- Primary Math Standards Edition 5A and 5B.

- Primary Math Standards Edition 6B.

- New Elementary Mathematics 1.

All of these textbooks can be ordered from the website SingaporeMath.com.

The format of this booklet is straightforward. Each section of *Elementary Geometry for Teachers* is listed. Any needed changes to the text of EGT for that section are listed immediately after the section name. For about half of the sections, the homework set in this booklet completely replaces the one in EGT. The remaining sections, such as Section 2.5 below, require no changes at all, so the homework problems can be done directly from EGT. **In this booklet, all page number references to the Primary Math textbooks refer to the Standards Edition**.

The authors thank Ben Ellison and Dan McGinn for skillfully and meticulously checking each homework problem and replacing the page number references from the U.S. edition with references to the Standards edition. We also thank the staff at Sefton-Ash Publishing, and Dawn and Jeffery Thomas at SingaporeMath.com for their support in the creation and distribution of this booklet.

Finally, a note for instructors. The revisions here are intended for the convenience of teachers whose schools use the Standards Edition. In college courses for pre-service teachers, and in professional development settings, the U.S. Edition of the *Primary Mathematics* series remains the recommended series for use with *Elementary Geometry for Teachers*.

Scott Baldridge

Thomas Parker

May, 2010

Learning to Measure

1.1 Measurement Problems

Homework Set 1

1. Using your ruler, draw a segment 4 cm long, one of 10 cm long, and one 20 cm long.

2. Measure your 10 cm segment in inches. It is roughly _____ inches long. Therefore, one inch is approximately _____ cm long.

3. Mark 5 points on your paper, making sure that no three of them are collinear. Label the points using letters A to E.

    a) Draw $\overline{CE}$.

    b) Draw $\overrightarrow{BA}$.

    c) Draw $\overleftrightarrow{BD}$.

    d) Name the intersection of $\overrightarrow{BA}$ and $\overrightarrow{AB}$.

4. Give "Teacher's Solutions with bar diagram and units" to Problems 6 and 8 on page 87 of Primary Math 4A. (Use the bar diagrams on the preceding pages as guides.)

5. Similarly, give a Teacher's Solution to Problem 22 on page 109 of Primary Math 4A.

6. Give Teacher's Solutions with bar diagram and units to Problems 8, 9, and 11 on page 98 of Primary Math 5A. Read pages 99–102 to get ideas on how to create bar diagrams for these problems.

7. Give a Teacher's Solution to the following problem.

Segment $\overline{AB}$ is 4 times as long as segment $\overline{CD}$. Segment $\overline{EF}$ is 3 cm longer than $\overline{CD}$. If $\overline{EF}$ is 8 cm long, how long is $\overline{AB}$?
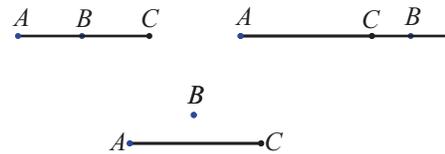
8. Examine the pictures above. Write a precise definition of the term "midpoint" by completing this sentence: "B is the **midpoint** of segment $\overline{AC}$ if . . . ."

9. How many lines can be drawn through distinct points P, Q, R, and S if

    a) P, Q, and R are collinear?

    b) no three of the points are collinear?

10. If you are given 5 points in a plane, no three of which are collinear, how many lines can be drawn through pairs of those points?

    Hint: Be organized! From each of the 5 points, draw a line to every *other* point; in the end you will have drawn each possible line *twice* – once from each of its endpoints.

1.2 Measuring Length

1. Make a 1-meter "tape measure" as follows:

   a) Cut an (unlined) sheet of paper into strips about an inch wide and tape the strips together end-to-end.

   b) If necessary, cut lengthwise again to give your long strip one straight side.

   c) Using your ruler, draw marks at 1 cm intervals along the straight edge up to the 1-meter mark. Make every tenth mark longer to indicate decimeters, and label these $10, 20, 30 \cdots$

   d) Color your tape measures (e.g. using a highlighter) in two alternating colors in 1-decimeter strips as pictured below.

2. (*Study the Textbook!*) Read Primary Math 3B pages 8–12, doing the problems in your text as you go and writing answers to the following questions on your HW paper.

   a) Problem 10 on page 10 asks students to order lengths. Write down (in one sentence) a 2-step process for determining such orderings.

   b) On page 12, the shaded regions show the mental math method "first add (or subtract) _____, then add (or subtract) _____".

   c) On page 19, answer Problems 1–5. Then answer Problem 21 on page 29.

3. (*Study the Textbook!*) Continue in Primary Math 3B, reading pages 13–18 and doing the problems in your text as you go. Then on page 19, answer Problems 6–9 and answer Problem 22 on page 29.

4. (*Study the Textbook!*) Continue, reading and doing the problems on pages 20–24 and page 26. Then on page 25 answer: Part (b) only for Problems 1–4, parts (a), (d) and (e) for Problems 5–7.

5. (*Study the Textbook!*) On page 24, Problem 8b uses which interpretation of subtraction: take-away, whole-part, or comparison?

6. Write a Teacher's Solution (with a bar diagram and units) for Problems 6 and 8 on page 103 of Primary Math 5A.

7. Conversions between metric units are done simply by shifting the decimal point. Make the following conversions.

   a) 867 cm = _____ m

   b) 532 mm = _____ cm

   c) 63.2 m = _____ mm

   d) 2.35 km = _____ m

8. How tall are you in meters and centimeters? (Make a mark on the wall and use your homemade tape measure.)

9. It takes about 10 minutes to walk 1 kilometer. A car on the highway goes about 100 kilometers in an hour. Roughly,

   a) how many kilometers is it across campus?

   b) how many kilometers to your dorm or apartment?

   c) how many kilometers is it to another city? (Pick a particular city and name it in your answer.)

10. If one side of a triangle is 6 cm and another is 10 cm long, the third side must be shorter than _____ cm and longer than _____ cm. (Hint: Draw several possible triangles before answering.)

11. A point B is said to be *between* points A and C if B is on the segment $\overline{AC}$.

    If B is between A and C, what equality involving the distances AB, BC, and AC must be true? Draw a sketch.

12. A point B is called a **midpoint** of a segment $\overline{AC}$ if B is between A and C and $AB = BC$.

    a) Draw a segment $\overline{AC}$ and mark the midpoint.

    b) Does it make sense to speak of a "midpoint of a line $\overleftrightarrow{AC}$"? Why or why not?

13. A line that intersects a segment at its midpoint is called a **bisector of the segment**.

    a) Draw a segment $\overline{AB}$ and two different bisectors.

    b) Does it make sense to speak of a "bisector of a line $\overleftrightarrow{AB}$"? Why or why not?

    A *counterexample* to a mathematical statement is an example where the statement is false. To prove that a statement is true, one must prove it true in *all cases*, but to show that it is false requires *only one* counterexample. Counterexamples are useful for teaching. Use this idea of *instructional counterexamples* to answer the following question.

14. Kevin thinks that the definition of midpoint can be shortened to this:

    A point Q in the plane is the midpoint of a segment $\overline{PR}$ if $PQ = QR$.

    Show that Kevin's definition is incomplete by drawing a counterexample.

1.3 Measuring Weight and Capacity

Changes to text: Page 15, line 1 should read "pages 30–45" and page 16, line 5 should read "pages 48–53".

Homework Set 3

1. (*Study the Textbook!*) Read Primary Math 3B pages 30–35, doing the problems in your text as you go.

   a) In Problem 1 on page 31, the scale changes from a 4 kg scale in parts (a) and (b) to a 2 kg scale in parts (c) and (d). Why did the textbook do that?

   b) On page 32, students begin doing what in Problem 3? Then, in Problems 5, 8, 10, 11, students begin doing what?

   c) On pages 30–35, which problem(s) ask students to order or compare weights?

   d) Problem (b) on page 34 can be solved by "counting down" (see [EMT] page 21). Illustrate this using a "jumps on a number line" picture.

2. (*Study the Textbook!*) Primary Math 3B continues developing arithmetic with weight units by giving word problems on pages 36–40.

   a) Read pages 36–39 and write the answers to Problems 2, 3, 4, 5, 7 on your homework sheet.

   b) On page 40, give Teacher's Solution (with bar diagrams similar to those on pages 36–38) for Problems 5–7.

3. Continue, reading pages 41–45. Answer Problems 3b and 5b on page 42, Problem 13mnop on page 47, and give Teacher's Solutions to problems 11, 13 and 14 on page 45.

4. Give a Teacher's Solution to Problem 4 on page 146 of Primary Math 5A.

5. (*Study the Textbook!*) Return to Primary Math 3B and read pages 48–56, doing the problems in your text as you go. This is the grade 3 introduction to capacity.

   a) What skill is required to answer the questions on page 48?

   b) Pages 51 and 52 give activities and exercises that show students that odd-shaped containers have definite capacities which can be measured by . . . by doing what?

   c) On page 53, students begin doing what in Problem 10? Then, in Problems 11–13, students begin doing what? Notice how quickly this lesson moves into arithmetic with units; that is possible because students have done similar exercises with length and weight.

   d) On pages 48–56, which problem(s) ask students to order or compare capacity?

   e) Problem (b) on page 55 can be solved by "counting up" (again, see [EMT] page 21). Illustrate this as "jumps on a number line".

6. In Practice A on page 57 of Primary Math 3B, answer Problems 1c and 1f, 2c, and 3abc.

7. (*Study the Textbook!*) Primary Math 3B continues developing arithmetic with units through word problems.

   a) Answer Problems 4–6 on page 57, and Problem 17 on page 65.

   b) Give a Teacher's Solution to Problem 16 on page 64.

   c) Which of the following operations with metric units is NOT done in these Problems: a) addition b) subtraction c) multiplication d) division e) ordering f) multistep.

8. Continue, reading pages 58–61. On page 62, answer Problems 1b, 2b, 3b, 4a, and 6, 7, and 8.

9. In Primary Math 5B, give a Teacher's Solution to Problem 18 on page 43.

10. In Primary Math 5B, give a Teacher's Solution to Problem 14 on page 75.

11. Complete the following expressions.

   a) 3.42 metric tons = _____ kg = _____ g.

   b) 1978 g = ___ kg = _____ metric tons.

   c) 758 $m\ell$ = _____ ℓ.

   d) 70 kg = _____ metric tons.

   e) ✳ In Primary Math 6B, give a Teacher's Solution to Problem 5 on page 112.

1.4 Measuring Angles

Changes to text: On page 19, line 3 should read "page 130" and line 4 should read "page 13".

Homework Set 4

1. For the times shown, find the indicated *reflex* angle between the clock hands in degrees (without using a protractor).

(a)	(b)	(c)
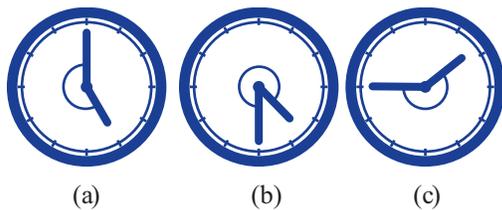
2. Through how many degrees does the earth revolve on its axis each hour? Each minute?

3. The distance from the equator to the north pole is almost exactly 10,000 km. Roughly, how many kilometers is:

   a) the distance around the earth at the equator?

   b) 1 degree of longitude?

4. (*Study the Textbook!*) An introduction to angles for third graders is given in Primary Math 3B, pages 127–131. Read these pages, doing the problems in your textbook as you read (not to be handed in) and answer the following questions.

   a) On these pages, students do which of the following?

      (i) compare angles (larger, smaller).

      (ii) count the number of angles in a figure.

      (iii) measure angles in degrees.

      (iv) identify right angles.

      (v) learn that right angles measure 90°.

   b) One must measure carefully to determine whether an angle is a right angle; it is not enough for it to *look* like a right angle. Which of the angles (a)–(f) on page 130 leads students to realize that?

5. (*Study the textbook!*) Read Primary Math 4A pages 112–115.

   a) Does the Primary Math curriculum immediately define a right angle as 90°? In the 3B book, how do student check whether an angle is a right angle?

   b) In Primary Math 4A, the very first picture on page 112 gives the definition of a degree in a subtle way. How is it defined?

   c) On page 130 of Primary Math 3B and on page 115 of Primary Math 4A (Problem 8), the angles open in many different directions. Why did the writers of the textbook do this?

6. (*Study the textbook!*) Now do all problems on pages 112–115 of Primary Math 4A as you read (do not record your answers in your HW) and answer the following questions.

   a) An angle can be thought of either as an amount of rotation or as a geometric figure formed by two rays and a small arc. For each of the problems 1–5 on pages 113 and 114 indicate which interpretation is being used (write "rotation" or "figure" for each).

   b) What issue is addressed in Problem 5 (bottom of page 114) that did not occur in the previous problems?

   c) Examine Problem 6 carefully; it explains two methods for measuring ∠p. Write a one or two sentence explanation of both methods.

7. (*Study the textbook!*) Primary Mathematics curriculum uses letters (like ∠a) to represent numbers (prealgebra) for the first time in the curriculum in Primary Math 4A. They do so here because there is a visual link between the angle (Angle a) and its measure ∠a. How many different letters are used on pages 112-115 of Primary Math 4A?

8. Using a straightedge (e.g. your ruler) and a protractor, draw angles of 47°, 135° and 291°.

9. (*Study the textbook!*) Read Section 9.1 in NEM I (pages 231–237). Answer the following questions as you read

   a) Does this section assume that the students already are experienced at measuring angles in degrees, or is that taught here?

   b) This section introduces two ways of writing angles: angles ∠ABC and ∠p can also be written how?

   c) Write down the definition of *reflex angle* given in this section.

   d) In this text, the sides of an angle are also called the _____.

   e) Protractors usually have two scales. The text describes the appropriate scale to use in a very clear 10-word phrase. Write down that phrase.

10. *(Study the textbook!)* In NEM1 at the bottom of page 235 shows students (and teachers!) four common mistakes (the last of these uses the word "produced" in the British sense, meaning "extended"). For each error, give a one sentence description and draw a sketch. Make your sketch different than the pictures in the book.

11. Continuing in NEM1, in Problem 4 at the top of page 237, do Parts (a)–(e) and the version of part (f) below. Here is some guidance, of the sort that teachers are expected to provide, for each part:

(a) Use a protractor.

(b) and (c) Use protractor and straightedge.

(d) This is asking you to extend the *ray RQ* past *Q* to some point that you name *T*.

(e) Use protractor and straightedge as before.

(f) What is the measure of $\angle SQU$? Can you explain this?

Geometric Figures

2.1 Fundamental Geometric Ideas

Changes to text: Page 29, line 3 should read "pages 78–81 in Primary Math 5B."

Homework Set 5

1. For each of the following times of day, sketch a clock face showing that time and find the measure (in degrees) of the angle formed by the hour hand and the minute hand. (Remember that the hour hand moves 30° an hour.)

    (a) 3:00   (b) 3:30   (c) 10:30   (d) 2:45.

2. In the figure, $a = 38°$. Find b, c, and d.
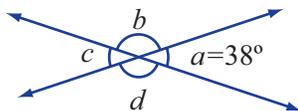
3. On page 238 of NEM1, do Problem 1 of Class Activity 2.

4. (*Study the textbook!*)   Read the rest of Class Activity 2. These exercises have students accept three basic facts after checking them in one example. These are not proofs; the purpose of the activity is to make clear *what the facts mean* and help make them evident to the students (rather than being teacher-announced truths).

    a) Copy down the three facts with their abbreviations. We will shorten the first two abbreviations to "∠s on a line" and "vert. ∠s".

    b) Are these the same three facts introduced on pages 78–89 of Primary Math 5B?

5. In Exercise 9.2 on pages 240-241 of NEM1, do problems 1b and parts (b), (f), (h) and (j) of Problem 2. Write your solutions in the manner of Worked Examples 1 and 2 on pages 239 and 240 on NEM 1. Be sure to include reasons in parentheses.

6. (*Study the textbook!*)

    a) Read pages 116–121 of Primary Math 4A. Does the book define the terms perpendicular and parallel, or does it just show examples?

    b) Now read pages 242–243 of NEM1. Does NEM1 define the terms perpendicular and parallel?

    c) Give a one or two sentence explanation of how Problem 2 on page 118 of Primary Math 4A helps students make sense of the seventh grade definition of perpendicular.

    d) Give a similar explanation of how Problem 2 on page 121 helps students make sense of the seventh grade definition of parallel lines.

7. On page 243 of NEM1, do Problem 1 of Class Activity 3 using a ruler and a set square. Then write a precise definition for the term *perpendicular bisector*.

8. (*Common Student Error*) When asked to find x in the figure below, Mary writes "$x = 50$" and explains "because

vertical angles are equal". What is x actually? What erroneous assumption did Mary make?
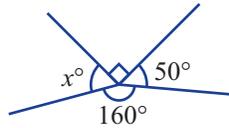
9. *(Common Student Error)* In the figure below, Jerry claims that "$a + b = 180$" stating as the reason, "∠s on a line." There may be several reasons why Jerry is making this error. He may not understand angle measurements or how to add angle measurements, he may not understand that to apply this fact the angles must be ⎯⎯⎯⎯⎯, or he may not understand that there are ⎯⎯⎯⎯⎯ degrees in a straight angle.
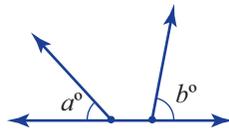
10. Draw a line L and a point P not on L. Using a ruler and set square, follow the procedure of Example 1.7 in this section to draw a line through P parallel to L.

11. A teacher asks her students for a precise definition of the term *circle*.

   a) Sarah says "A circle is a round segment with no endpoints". Name at least two things wrong with Sarah's definition.

   b) Michael says "A circle is 360°". What two notions is Michael confusing?

   c) Write down a precise definition of the term *circle*.

12. Practice drawing circles with a compass until you can draw complete circles without stopping. Then on your homework paper draw circles of radius 2 cm, 6 cm and 10 cm, using your ruler to set the compass width.

13. Use your compass to draw a circle; label your circle C and its center O. Choose any point on the circle and label it P. Draw another circle of radius OP with center P.

14. Use your compass to draw circle B; label its center 0. Choose any point on the circle and label it Q. Draw two more circles of different radii such that both circles intersect circle B only at point Q. (Hint: Draw line $\overleftrightarrow{OQ}$ first.)

2.2 Triangles

Changes to text: On page 38, lines 4–6 should read "read pages 82–83 ... on pages 82–83."

Homework Set 6

In this homework set you will study two presentations of the facts about the angle sum of a triangle: one for 5th graders and one for 7th graders.

1. (*Study the Textbook!*) The fact that the sum of the measures of the angles of a triangle is 180° is introduced in pages 82–85 of Primary Math 5B.

   a) Do the experiment described on page 82. Is this a "picture proof" as defined in this section?

   b) The statement (at bottom of page 82) abbreviates language in the interest of clarity. Use the words "measure" and "interior" to expand this statement to make it complete and correct.

   c) Using a protractor, measure the angles of triangles A and B in Problem 1 on page 83. What do you get for the sum of the interior angles?

   d) What do students gain by doing this problem? (Hint: read this section!)

2. (*Study the Textbook!*) Continue reading and doing the problems in Primary Math 5B pages 83–85. Notice that the students are no longer measuring angles. Instead they are *figuring out angles using the given information*.

   a) Answer Problems 2 and 3 on page 83.

   b) Do the folding experiment described on page 84. Again, according to this section, what is the purpose of such picture proofs?

   c) Answer Problems 5 and 6. Write the answers as a list of numbers.

d) Here is another 5th grade explanation of the state-ment before Problem 5 on page 84. Justify the first step by filling in the blank using the abbreviation for one of the boxed facts in this section
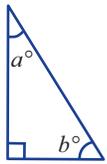
The figure shows a right triangle.
To Prove: $a + b = 90°$.

$a + b + 90 = 180$ _____

Therefore $a + b = 90$.

3. Find the value of c in the figure.
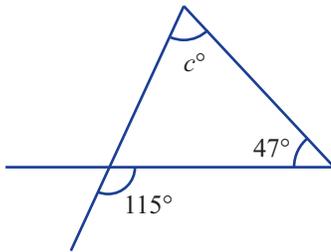
4. In the figure, $\overline{AB}$, $\overline{CD}$ and $\overline{EF}$ are segments. Find the value of d.
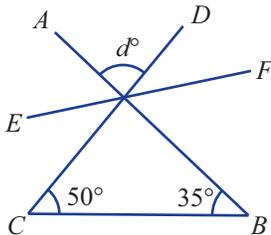
5. Find the value of e in the figure.
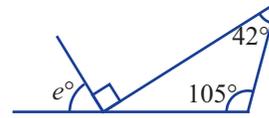
6. Find the value of f in the figure.
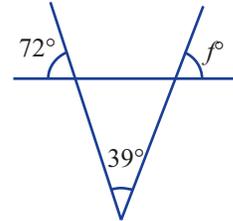
7. Find the value of g in the figure.
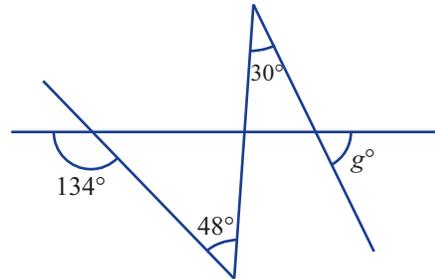
8. (*Study the textbook!*)   In NEM, read pages 262-263 and study the solution to Worked Example 4 on page 263. Then in Exercise 10.2 (pages 263-266):

   a) Write solutions to Problems 3a, 3c, 3h, and 3i using the format of Worked Example 4. Be sure to include reasons in parentheses.

   b) Use the same format to write solutions to Problems 4a, 4h, and 4l.

## 2.3 Symmetry and Triangles

Homework Set 7

In this homework set you will study the presentation of the facts about angles of a triangle, first at the grade 5 level, then at the grade 7 level.

1. (*Study the Textbook!*) The basic facts about equilateral and isosceles triangles are introduced in Primary Math 5B. Read pages 86–89 carefully and answer the follow-ing questions.

   a) The fact that the base angles of an isosceles triangle are congruent is illustrated how?

   b) Read Problem 2 on page 87 carefully. This problem illustrates the fact that "if a triangle has two equal angles then _____".

   c) Write the answers to Problems 3 and 4.

   d) Problems 5, 6 and 7 are 2-step problems that com-bine facts about isosceles triangles with one other fact. For each, write the answer and state the "other fact" used.

   e) Write the answers to Problem 8.

2. Find the value of x in the figure.
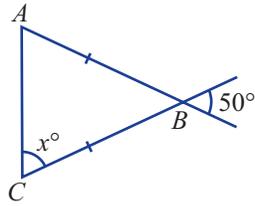
3. The figure shows a pentagon whose central angles are all equal: $a = b = c = d = e$. Find the value of y and the value of z.
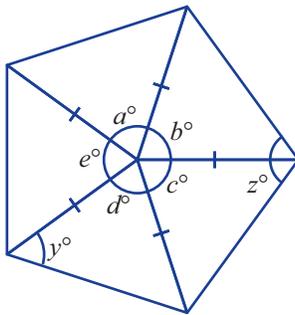
4. Find the value of r and of s in the figure.
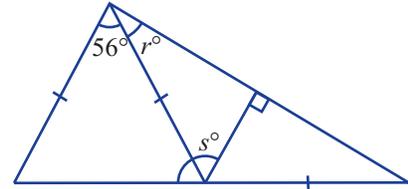

5. (*Study the textbook!*) Read Section 10.3 in NEM1, pages 268–270. Do Problems 1, 2a, 2b, 3a, 3b, 3c, 3d, and 3g.

6. (*Study the textbook!*)   Read Sections 11.1 and 11.2 in NEM1 pages 290–298. Do problem 4 on page 297 by drawing the lines of symmetry in your NEM1 textbook (do not copy to your homework paper). Write your answer to part (xii) on your homework paper.

7. Do Problem 6 on page 298 of NEM1.

2.4 Parallelograms, Rhombuses and Trapezoids

Homework Set 8

1. To stay in practice, make the following conversions.

    a) 18 m 67 cm = ——————— m
    b) 1 km 230 m = ——————— m
    c) 86 mm = ——————— cm
    d) 4 m 7 cm = ——————— cm
    e) 23.8 cm = ——————— mm

2. This exercise provides practice in creating "instructional counter-examples". Draw:

    a) a rectangle that is not a square.
    b) a parallelogram that is not a rhombus.
    c) a kite that is not a parallelogram.
    d) a trapezoid with two opposite sides that clearly have different lengths.
    e) a parallelogram whose diagonals are clearly not perpendicular.

3. Is this figure a kite? (Read the definition of kite carefully).
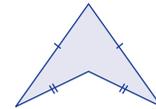
4. (*Study the Textbook!*) The basic facts about parallelograms and rhombuses are introduced on pages 95–98 of Primary Math 5B.

    a) Write down precise definitions for *parallelogram*, *rhombus* and *trapezoid*.
    b) What fact is illustrated on page 96? (see the pictured "student helper"). The illustrations show how to match up angles by cutting, ———— , and sliding.
    c) What fact is illustrated on page 97?
    d) Write answers for Problems 3 and 4.
    e) Problem 5 uses the fact established in Problem ——, now applied to a trapezoid instead of a parallelogram.
    f) Copy the picture in Problem 5 and show how to cut and slide to see each of the two facts needed to solve the problem. Can you show both in one picture?

g) Write the answers to Problem 6.

5. Read pages 99–102 of Primary Math 5B. These give students practice in geometric constructions with ruler, set square, and compass. Using those tools, do the constructions called for in Problems 3 and 5 on page 102.

6. Now go to Primary Math 5B. Work through pages 60–65, answering Problems 1–11 in your book, then copying the answers onto your HW paper. Take time to understand the pictures. Notice how the questions take you through the intermediate steps *in order*.

7. Continuing in Primary Math 5B, work through pages 66–68, answering Problems 1–5.

8. Continuing, answer Problems 1–7 of Practice B on pages 69–70.

9. Draw a kite and label the vertices A, B, C, D.

a) Draw and name a line of symmetry.

b) Describe a folding exercise that shows this symmetry.

c) The symmetry shows which two angles are equal?

d) The line of symmetry bisects which two angles?

10. In general, how many lines of symmetry do each of the following figures have? Draw sketches showing the lines of symmetry.

a) isosceles triangle

b) kite

c) rectangle

d) parallelogram

e) square

f) circle.

11. In general, does a parallelogram have rotational symmetry of order 2? of order 4? (see page 42) Draw a sketch.

2.5 Geometric Constructions

No changes to either the text or the homework.

Finding Unknown Angles

3.1 Unknown Angle Problems

Changes to text: On page 56, line -9 should read "presented in Primary Mathematics 5B", and line -2 should read "pages 78–81 in Primary Mathematics 5B". On page 57, lines 7–8 should read "pages 82–89". On page 58 line 5 should read "(pages 95–98)".

Homework Set 10

1. (*Study the Textbook!*) Give one-sentence answers to Exercises 1.1, 1.2, and 1.3 in this section.

2. (*Study the Textbook!*) Turn to page 80 of Primary Math 5B.
   - Do all of the problems on pages 80 and 81. Write your answers as a single list of numbers, separated by commas, but *not* labeled by the problem numbers.
   - Note the variation: on pages 80 and 81, how many different letters are used to stand for numbers?

3. On pages 85–86 of Primary Math 6B, answer Problems 5, 6, and 8.

4. Answer Problem 2 on page 264 of NEM1. Show your reasoning.

5. For Problem 3 on page 264 of NEM 1, write Teacher's Solutions to parts b, d, e, g, and j.

6. For Problem 4 on the next page (page 265) write Teacher's Solutions to parts b, c, f and g.

7. (*Study the Textbook!*) Compare the grade 6 and grade 7 problems you did in this HW Set. Name two new features required in solving the grade 7 problems that are not present in the grade 6 problems.

3.2 Finding Angles Using Parallel Lines

3.3 Angles of a Polygon

No changes to either the text or the homework in Sections 3.2 and 3.3.

Deductive Geometry

4.1 Unknown Angle Proofs

## 4.2 Congruent Triangles

4.3 Applying Congruences

No changes to the text or homework for Sections 4.1, 4.2 and 4.3.

4.4 Congruences in Quadrilaterals

**Changes to text:** page 96, line 2 should read "(page 95 of Primary Math 5B)."

Homework Set 16

1. Do Problem 2 on page 282 of NEM1. Copy only the letters for the properties (you don't need to write the sentence). For example, the first line of the table should start: "(a) T F T …".
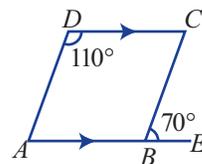

2. Do all parts of Problem 3 on pages 283-284 of NEM1. Do not give proofs or Teacher's Solutions —just find the unknowns x and y in each figure.

3. Prove that $ABCD$ is a parallelogram.

4. In the figure, $\overline{SR} \mathbin{/\!/} \overline{PQ}$, $PS = SR$, and $PQ = QR$. Prove that $PQRS$ is a rhombus, i.e., prove that $PS = SR = PQ = QR$.

5. In the figure, $ABCD$ is a parallelogram, and $\overline{AE}$ and $\overline{CF}$ are perpendicular to $\overline{BD}$. Prove that $AE = CF$.
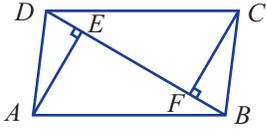
6. In the figure, $ACDF$ is a rectangle and $BCEF$ is a parallelogram. Prove that $\triangle ABF \cong \triangle DEC$.
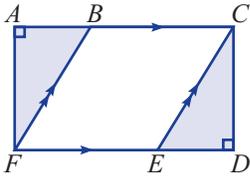
7. In the figure, $ABCD$ is a rectangle, and $AP = CR$ and $AS = CQ$. Prove that $PQRS$ is a parallelogram.
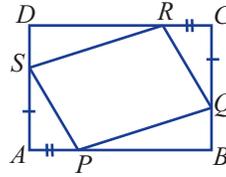
8. Give an Elementary Proof: the diagonals of a square are perpendicular. Start by drawing a picture and writing:

   Given: Square $ABCD$ with diagonals $\overline{AC}$ and $\overline{BD}$
   To Prove: $\overline{AC} \perp \overline{BD}$.

9. Prove that a parallelogram with one right angle is a rectangle.

4.5 Transformations and Tessellations

Homework Set 17

1. In this problem you will determine which regular n-gons tessellate. The steps outline a typical classroom exercise.

   a) Recall that the sum of the interior angles of any n-gon is $(n - 2)180°$. Use this fact and a sketch to explain why the measure of each interior angle for any n-gon is

   $$I_n = 180° - \frac{360°}{n}.$$

   b) List all the factors of 360 greater than or equal to 60.

   c) Fill in the following table.

# of sides	Interior angle	Factor of 360°?	Tessellate?
3	60°	✓	✓
4			
5		✗	
6			
7			
8			
9			
10			

   d) Use the formula for I_n and some algebra to show that whenever n is bigger than 6, I_n is between 120° and 180°.

   e) Explain why your answers to parts a)-d) show that regular n-gons tessellate only for $n = 3, 4$ and 6.

2. Study the last picture of this section. Follow the same procedure to make two simple tessellations. Draw at least 6 tiles for each. Coloring may make your picture clearer.

3. Do the following constructions.

   a) Draw a triangle $\triangle ABC$ and a ray $\overrightarrow{DE}$. Show how to move the triangle $\triangle ABC$ 4 cm in the direction $\overrightarrow{DE}$ using only a compass, ruler, and a set-square.

   b) Draw a triangle $\triangle ABC$ and a line l. Show how to reflect the triangle $\triangle ABC$ across the line l using a compass and straightedge.

   c) Draw a triangle $\triangle ABC$ and mark a point D. Show how to rotate the triangle $\triangle ABC$ about the point D an angle measure of 45°, using a compass and straightedge.

4. (Study the textbook!)   Read pages 77–80 in Primary Math 4B. Use graph paper to show how Shape B on page 79 can tessellate (graph paper can be obtained from www.printfreegraphpaper.com).

5. (Study the textbook!)   Assuming that 1 to 2 pages can be covered in a day's lesson, estimate how many class days fourth grade Primary Math teachers spend on tessellations.

6. Explain, with pictures, why *every parallelogram tessellates*.

7. Explain why *every triangle can tessellate* (this can be done with one sentence and one picture using your answer to Problem 6).

8. ❋ This problem takes you through a classroom explanation of the fact that *every quadrilateral tessellates*.

   On a sheet of thin cardboard, draw a quadrilateral about 3 inches across. Make the sides straight (use a ruler) and make the four interior angles have clearly different measures (your quadrilateral needn't be convex). Label the interior angles w, x, y, z in clockwise order.

   a) What is the sum $w + x + y + z$ of the interior angles?

   Cut out your quadrilateral with scissors; this is your "template". On a large blank sheet of paper, make a copy of your quadrilateral by tracing around your template. Call this figure "Tile 1" and label its interior angles w, x, y, z as on the template. Draw additional tiles by repeatedly applying the following step (which is named to indicate the edge we fill across).

   STEP xy: Without flipping the template, align the template along the edge between interior angles x and y of one of the previously-drawn tiles so that y on the template matches with x on the tile and vice versa. Trace around the template to create a new tile. Label the interior angles of the new tile w, x, y, z as on the template.

   Do STEP xy to draw Tile 2. Then do STEP wx on Tile 1 to draw Tile 3.

   (b) Look at the vertex of Tile 1 labeled by angle x. It is surrounded by 3 tiled angles and an un-tiled angle. The un-tiled angle has the same measure as which interior angle of your template? Why?

   (c) Fill in the blanks: one can fill in this un-tiled angle either by doing STEP ___ on Tile ___ or by doing STEP ___ on Tile ___. Do these give the same result?

   (d) After drawing Tile 4 by your first answer to (c), explain why the paired sides of Tiles 2 and 4 have equal length.

   (e) Similarly, explain why the paired sides of Tiles 3 and 4 have equal length.

   (f) Draw at least 6 tiles. Can this tiling procedure be continued indefinitely?

Area

5.1 Area Units

Changes to text: One page 110, line 6 should read "pages 139 to 146" and line 11 should read "at the end of Primary Math 4B".

Homework Set 18

Work with area starts early in the Primary Mathematics curriculum. Students find areas by counting unit squares in grade 2. In Primary Math 3B, area units are defined and used. Area and perimeter are introduced in the same set of lessons, bringing out the similarities and the differences between length and area.

1. (*Study the Textbook!*)   Read pages 139–146 in Primary Math 3B. In the first exercise, students use a "nonstandard unit" — a square card of unspecified size.

   a) After that, which standard units are introduced?

   b) How many class days are spent on this material? (The arrows at the bottom of the pages refer students to 1-night homework exercises in the Workbooks.)

2. (*Study the Textbook!*) In Primary Math 3B, answer Problems 4 and 7–11 on pages 139–146. What is the point of Problem 4 on page 142?

3. Draw — actual size — a square of size 1 cm by 1 cm and a square of size 1 in by 1 in.

4. (*Study the Textbook!*) Read pages 147–149 in Primary Math 3B.

   a) Write down the 3rd grade definition of perimeter given here.

   b) The three examples on page 147 all have the same perimeter. What point does this illustrate?

   c) What practical difficulties will students have doing Problem 1 on page 148? (If you aren't sure, try doing the problem!)

   d) After doing Problems 3–5, students should understand what fact about the relationship between area and perimeter?

5. (*Study the Textbook!*) Page 141 of Primary Math 4A illustrates the formula for the area of a rectangle whose sides have whole number lengths. Examine the pictures carefully. Describe how this sequence of pictures evolves: what is the difference between rectangles A and B and rectangle C?

6. (*Study the Textbook!*) Continuing on pages 141–144, what is the difference between the rectangles in Problem 4 on page 143 and Problem 1 on page 144? Rectangles A and E in Problem 1 on page 144 are tilted; what do students learn from this?

7. Do all problems in Practice A on page 144 of Primary Math 4A.

8. In terms of area, the commutative property can be in-

terpreted as the statement: *The area of a rectangle is Length × Width and is also Width × Length.* What is the interpretation of the distributive property in terms of area? (See the picture in this section.)

9. Use the area model to justify the statement that $2\frac{1}{2} \times 3\frac{1}{4} = 8\frac{1}{8}$. (Draw a diagram and show the reasoning).

10. Recall that one decimeter is 10 centimeters (1 dm = 10 cm). Draw — actual size – a square of size 1 dm by 1 dm.

11. Draw a rectangle 1.5 dm by 0.7 dm (actual size). Give a

series of pictures and 1-sentence instructions for a class that show how to calculate the area in square decimeters (dm^2) by the following steps. *Suggestion: look over pages 331-332 of NEM1 before starting.*

a) Divide length and width into centimeters and make a grid of 1 cm squares.

b) Find the area in cm^2 by multiplication.

c) Separately find the number of cm^2 in 1 dm^2.

d) Express the area of your rectangle in dm^2.

5.2 Rectangles and Area Properties

Changes to text: Page 115, line 2 should read "(see page 141 of Primary Math 4A)."

Homework Set 19

1. Do Problems 3–5 on pages 148–149 of Primary Math 3B mentally. What idea is taught in these problems?

2. (*Study the Textbook!*)   Answer the following questions about Primary Math 4A as you read and mentally do the problems.

   a) Page 145 recalls the meaning of the area and the _____ of a rectangle. Write down these definitions exactly as they are given.

   b) Answer problems 3–8 on pages 147–148. In which of these problems are we given the width and height and asked to simply calculate width × height?

3. (*Study the Textbook!*)   In Primary Math 4A, page 151 introduces the idea of decomposing figures into rectangles to find their area.

   a) Answer the two questions on page 151.

   b) If you know only the perimeter of a figure (without a picture), can you determine its area? Why or why not?

   c) If you know only the area of a figure (without a picture), can you determine its perimeter? Why or why not?

4. (*Study the Textbook!*)   Solve Problems 1 and 2 on page 152 (examine the method and write down 2 numbers for each). Note the hints given by the "student helpers" in the margin.

5. (*Study the Textbook!*)   On pages 151–152 of Primary Math 4A:

   a) Answer Problem 3.

   b) Do Problem 4 first by dividing the shaded region into 3 rectangles, then by the method described by the student helper in the margin. Which is easier?

   c) Answer Problem 5.

   d) Answer Problem 6.

6. In Primary Math 4A, answer Problems 1 and 2 on page 149 and all Problems in Practice C, pages 155–156.

7. (*Study the Textbook!*)   In Primary Mathematics 4A,

   a) Were there any problems that involved lengths or areas that were fractions or decimals?

   b) All measurements and answers involved which units?

8. Give a complete proof: In a parallelogram ABCD, the diagonal $\overline{AC}$ bisects the parallelogram into 2 congruent triangles $\triangle ABC$ and $\triangle CDA$.

9. Which two area properties allow you to conclude that

$$\text{Area}(\triangle ABC) = \frac{1}{2}\,\text{Area}(ABCD)?$$

10. Dana said that the shaded triangle has area 6 cm^2 because "it's half the area of the rectangle". Check *all* of the area properties she was implicitly using.

3 cm

4 cm
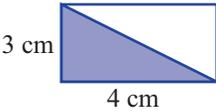
☐ area is ≥ 0

☐ $\cong$ figures

☐ areas add

☐ area of rect = $L \times W$.

11. Describe a rectangle whose perimeter is over 100 ft but whose area is less than 1 ft^2.

# 5.3 Area of Triangles, Parallelograms and Trapezoids

1. Draw an obtuse triangle. Show the three base-altitude pairs (make three pictures if necessary).

2. It is common for students to erroneously apply the area formula for the triangle by interpreting "height" to mean the length of one side.

   In fact, the height of a triangle is *never* equal to the length of a side unless the triangle is a _____ triangle.

3. (*Study the Textbook!*) Read pages 115–121 in Primary Math 5A.

   a) On page 116, in all three cases, the area of the triangle is related to which rectangle?

   b) In the third of the five pictures on page 66, the first arrow indicates how folding shows that two triangles are congruent. Draw the same figure on your paper, give the vertices letter names and give a formal proof that these two triangles are indeed congruent (ignore the bottom two pictures on this page). *Do not use the grid shown; the grid applies only for triangles with certain side lengths.*

4. (*Study the Textbook!*) Call the three cases shown on page 116 of Primary Math 5A, "Case 1", "Case 2" and "Case 3".

   a) For each of the four parts of Problem 4 on page 119, identify whether one should apply Case 1, 2 or 3.

   b) Answer Problem 5 on page 120.

   c) Answer Problems 6 and 7 on pages 120–121 .

   d) Answer all problems in Practice B on page 122.

   e) Which of the problems on page 122 is an example of i) finding the area of a complicated polygon, and ii) finding the area of a composite figure.

5. Sarah claims that, when deriving the formula for the area of a triangle, it is not necessary to consider Case 3 because for any Case 3 triangle one can draw an altitude that lies inside the triangle, and that was considered in Case 2. What's wrong with Sarah's reasoning?


just use this altitude!
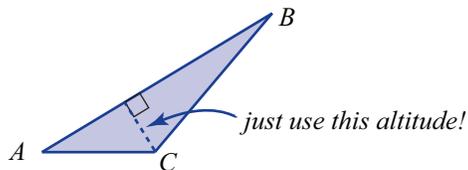
6. Write *Teacher's Solutions* to the four problems in Problem 28 on page 134 of Primary Math 5A. Make your solutions clear and complete and at the grade 5 level.

7. Do Exercises 3.2, 3.3 and 3.4 in this section.

8. Use the area formula for a triangle to obtain the area formula for a parallelogram by the approach suggested after Exercise 3.3 in this section. Write your answer in the "Teacher's Solution" format: draw and label a picture and give an explanation with equations and a few words.

9. Here are three methods for finding the area formula of a trapezoid. Show that all three methods lead to the same area formula. (In each picture, *a* and *b* are the base lengths and *h* is the height.)

   a) Divide the trapezoid into two triangles as illustrated below. Calculate the area of each triangle and add.
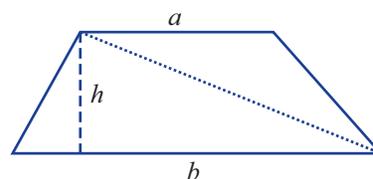
   b) Divide the trapezoid in half and put the top half next to the bottom half. Label and calculate the area of the parallelogram created.
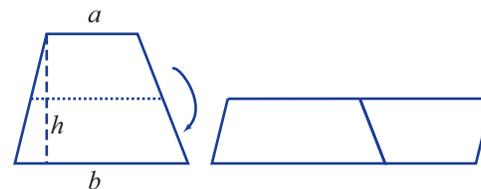

   c) Divide the trapezoid into two triangles and a rectangle. Calculate the area of each and add them together.
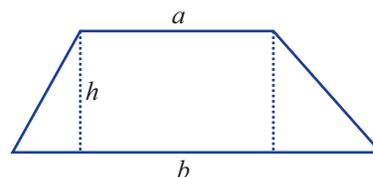

10. (*Study the Textbook!*) In NEM1, read Class Activity 3 at the top of page 334. Answer all the questions. Justify your answers to parts (c), (d) and (e).

11. Give Teacher's Solutions to Problems 12 – 15 on page 338 of NEM1.

12. Find the area of the shaded figures below.

a)

b)

c)
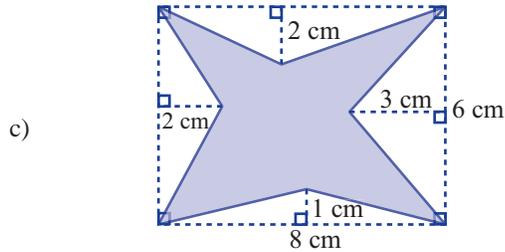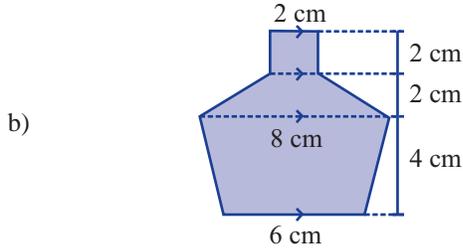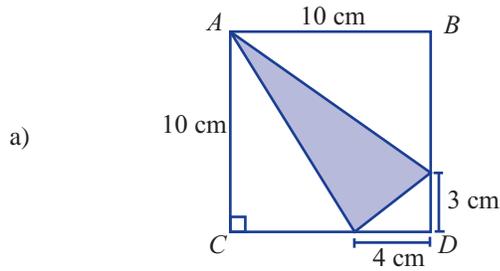
13. Using Example 3.6 as a guide, write a complete proof for the area formula for a parallelogram based on the idea that, in the picture below, Area(P) = Area(big rectangle) $-$ 2Area(S).
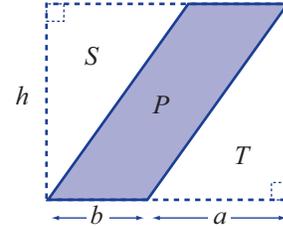
14. a) Use the fact that the diagonals of a kite are perpendicular to prove that the area of the kite is

$$\text{Area of Kite} = \frac{1}{2} cd,$$

where c and d are the lengths of the diagonals.

b) Explain (only 5 words needed!) why the same formula holds for a rhombus:

$$\text{Area of Rhombus} = \frac{1}{2} cd,$$

where c and d are the lengths of the diagonals.

c) If $\square ABCD$ is a square with side length s and diagonals of length d, use the formula from part b) to conclude that $d = \sqrt{2} \cdot s$.

Pythagorean Theorem with Applications

No changes to either the text or the homework in all of Chapter 6.

Similarity

7.1 Introducing Similarity and Similar Right Triangles

7.2 Similar Triangles

No changes to either the text or the homework in Sections 7.1 and 7.2.

7.3 Coordinate Systems and Slope

Homework Set 26

This homework set has three strands. Problems 2–6 examine how the Primary Mathematics Standards Ed. textbooks develop graphing ideas, beginning with pictograms and ending on the verge of defining slope. Problems 7–12 involve slope, and the remaining problems are map-scale problems.

1. Draw a number line and, without using a calculator, indicate the approximate location of the following numbers:

$$\frac{5}{3} \qquad 0.8 \qquad 2.04 \qquad -1.2 \qquad -\frac{31}{15}$$

2. (*Study the Textbook!*) Look over pages 101–113 of Primary Math 2B. There are three basic ways to represent data or relationships:

   a) As table of values.

   b) As a bar graph or line graph.

   c) As an equation, like $y = 3x + 5$.

   Which is taught first in the Primary Math curriculum, graphs or equations? Most curricula follow this order.

3. (*Study the Textbook!*) Page 108 of Primary Math 2B introduces bar graphs to students. Compare the pictogram

on page 104 with the bar graph on page 108. List four specific differences between these graphs.

4. Read pages 109–113 of Primary Math 2B, answering the questions mentally. How do the graphs on pages 106 and 110 of Primary Math 2B differ from the others on pages 102-113?

5. (*Study the Textbook!*) Read pages 119–121 in Primary Math 4B. We have moved from bar graphs to line graphs. Notice that:

   - The horizontal axis evolves to be a number line. The idea of graphing in the coordinate plane is being developed.

   - Most of the questions on pages 120 and 121 ask about changes in the height of the graph. These are really questions about slope.

Copy the graph from Problem 1 on page 120. Then draw two rise-run triangles that students should focus on to answer questions 1(a) and 1(b).

6. (*Study the Textbook!*) Page 121 gives the first example of a linear graph. Read it carefully and answer the questions mentally. Then write *three* aspects of this linear graph problem that make it especially easy for students to understand.

7. Show that the points $A(0,0)$, $B(2,1)$, $C(1,3)$ and $D(-2,4)$ are the vertices of a trapezoid (graph the points and calculate slopes).

8. Find the number z so that the line joining $A(z,4)$ and $B(2,3)$ is parallel to the line joining $C(-5,3)$ and $D(1,1)$.

9. For the points $A(1,7)$, $B(4,5)$ and $C(3,2)$:

   a) Find the slope of $\overline{BC}$.
   b) Find the point at which the line through A parallel to $\overline{BC}$ cuts the *y*-axis.

10. The vertices of a quadrilateral are the points $R(1,4)$, $S(3,2)$, $T(4,6)$, $V(2,8)$. Is the quadrilateral a parallelogram? Explain why or why not.

11. If the line containing points $(-8,m)$ and $(2,1)$ is parallel to the line containing $(11,-1)$ and $(7,m+1)$, what must be the value of m?

12. What values of k will make the line containing points $(k,3)$ and $(-2,1)$ perpendicular to the line through $(5,k)$ and $(1,0)$?

13. In the homework for the previous section, you did some of the "map scaling" problems on page 390 of NEM1. Now do one more: write a Teacher's Solution to Problem 4 on page 390.

14. Do Problem 6 at the bottom of page 399 of NEM1.

15. A floor plan for a house has a scale of 1 cm: 1.6 m.

   a) If a room is 3 cm by 4.8 cm on the floor plan, how large is the actual room?
   b) If the house is 18 m wide, how wide is it in the floor plan?

16. A model of the Statue of Liberty is 9 cm tall (including the pedestal and raised arm), while the actual statue is 300 feet tall. Give a Teacher's Solution to the following two questions. Be sure to include the units at each step.

   a) If the top of Liberty's head is 7.8 cm high on the model, how high is her head on the actual statue?
   b) If the granite pedestal is 90 feet high, how high should it be on the model?

17. On a map with a scale of 1 cm to 8 miles, what is the distance between two villages if they are:

   a) 24 miles apart,
   b) 83 miles apart,
   c) 18 miles apart,
   d) 1600 miles apart?

In Primary Math 6B, students learn to solve simple linear equations on pages 181-184, and then study their graphs on pages 182-192. This material was added especially for the Standards edition. It illustrates the fact that **textbook presentations are sometimes flawed**. *The following exercises illustrate how teachers sometimes have fix or enhance textbooks.*

18. Read page 185 of Primary Math 6B.

   a) Does the letter k play an essential role, or is it unnecessarily confusing?
   b) Rewrite the text above the chart on this page (including what the girl says) to eliminate k and just work with $y = 2x$.

19. Similarly, on page 186, (a) rewrite the text at the top of the page to eliminate k, and (b) rewrite the text at the bottom of the page to make (a) and (b) into *questions*.

20. Study page 187 of Primary Math 6B. The bottom example simultaneously introduces negative slope and fractional slope, neither of which are present in previous examples. Make up two new problems to insert between the two examples on page 187:

   a) One introducing fractional slope.
   b) A second one introducing negative slope.

7.4 Similar Right Triangles and Trigonometry

No changes to either the text or the homework.

Area Concepts and Circles

8.1 Converting Area Units and Scaling

8.2 Circles and Pi

Changes to text: Page 177, line -5 should read "(see page 128 of Primary Math 4A)".

Homework Set 29

1. The following numbers can be used as approximations of π. How accurate are they? For each, express the relative error as a percent (significant to 1%). Recall that relative error is the fraction (error)÷ (true value).

   a) 3   b) 3.14   c) 22/7   d) 3.1416   e) $\sqrt{10}$.

2. (*Study the Textbook!*) Work through pages 8–14 of Primary Math 6B, reading the text and answering all the problems (write the answers in your textbook).

   a) Make a list of the geometry terms (vocabulary words) that are introduced in these pages.

   b) Many of these problems take π to be either 3.14 or 22/7. Is there any problem here that *doesn't* do that?

3. Find the perimeters of all the shapes on page 19 of Primary Math 6B. Also find the perimeter of the shaded region in Problem 1 on page 26.

4. A piece of wire 16 cm long is wound around a circular pipe with a radius of 20 cm. What central angle does the wire subtend? (The wire may wrap around more, or less,

than shown in this picture). Express your answer in degrees.
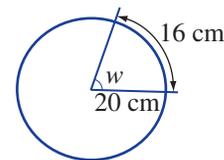
5. (*Study the Textbook!*) These ideas are revisited at the seventh grade level in NEM1 pages 339-341, along with the formula πR^2 for the area of a circle (which we will discuss in the next section).

   a) Read page 339. How is π defined here? Give your answer in words, not as a formula.

   b) In Exercise 12.2, do Problems 1e, 2b, and 3b, 4c, and 5a. Note that every problem is different! One cannot solve these by just memorizing formulas.

   c) If you ignore decimal points, many of the numbers in these problems are multiples of 7 (e.g., 28, 21, 154). Why did the authors choose multiples of 7?

6. Continuing in NEM1, read Worked Example 6 on page 341. Give Teacher's Solutions to Problems 7, 9, 10, and 11 on page 342.

7. A machine part is made by bending a metal strip into a circular arc according to the design below. How many such parts can be made from a metal strip 2 meters long?
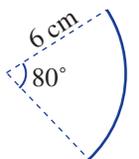
8. A flower is drawn inside a circle of radius 10 cm. The flower is made of arcs of radius 10 cm. Find the total perimeter of the flower.
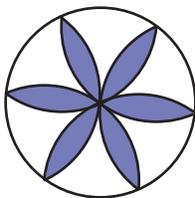
9. a) The circumference of a circle is 15 inches. To increase the circumference to 18 inches, how much longer must we make the radius?

   b) If the radius of a circle is increased by 2 units, how much does the circumference change? *Hint:* Let r and C stand for the original radius and circumference. Then $C = 2\pi r$ and the new circumference is $C_{new} = 2\pi(r + 2) = \ldots$.

   c) Suppose that a wire is stretched tightly around the Earth along the equator. (Assume that the Earth is a circle along the equator with a circumference of $40,000\ km$.) Approximately how much would the length of the wire have to be increased so that it was everywhere 1 meter above the surface?

10. (*Calculator*) The earth travels in an orbit which is nearly a circular path with the sun at the center and a radius of $93,000,000$ miles. If the earth makes one orbit in 365 days, what is its approximate speed in miles per hour?

8.3 Area of Circles and Sectors

Changes to text: Page 183, line -1 should read "Read Chapter 7". In the middle of page 185, the text refers to a picture in the Primary Math 6B U.S. edition is not included in the Standards Edition. A similar picture, appropriate for grade 5 or 6 students, is shown on page 25 below.

Homework Set 30

1. (*Study the Textbook!*) Primary Math 6B covers circumference and area of circles on pages 8–27. About how many class days is that? To get a rough count, count the number of arrows referring the students to Exercises in Workbook 6B and add a few because homework may not be assigned every day.

2. Do all the problems on pages 26–27 of Primary Math 6B. The students have been prepared to tackle some challenging problems!

3. If the diameter of a circle is doubled, how does its area change? Explain your answer in one sentence.

4. Find the ratio of the radii of two circles if the ratio of their areas is 4 : 9.

5. Find the radius of a circle whose area is equal to the sum of the areas of two circles of radii 5 cm and 12 cm.

6. Do Problem 9 on page 345 of NEM1. Which has larger area: the shaded or unshaded region?

7. A circle has radius 1 unit. What percent of its area lies outside the circle with the same center and a) radius $\frac{1}{2}$ unit?   b) radius $\frac{3}{4}$ unit?

8. Jessica used her calculator to find the area of a disk of radius 12 in. She first found that $\pi \cdot 12 = 37.7$, then squared this number and wrote Area $= 1421$ in^2. What was Jessica's error?

The grade 6 material on areas and perimeter is reviewed and extended on pages 339-348 of NEM1.

9. Do Problems 3a, 5b, and 6a on page 342 of NEM1.

10. Some grade 6-7 area problems involve introducing and solving for an unknown. On page 343 of NEM1, Worked Example 8 presents such a problem. Read it carefully, and give similar Teacher's Solutions to Problems 13, 14, 15, and 16 on pages 345-346.

11. Do Problems 17 and 18 on page 346.

Every good curriculum includes challenging exercises for students who finish early—exercises that also help teachers! In NEM1, the starred problems on pages 346-347 and the "Challenger" problems on 348-349 are such exercises.

12. ✳ Do Problems 19-21 on page 346.

13. ✳ Give Teacher's Solutions to Problems 22, 23 and 24 on page 347 (Hint for 24: use your answer to Problem 17 on the previous page.)

14. ✳✳ If you finish early, try *Challenger Problems* 1, 2, 3, and 5 on pages 348-349 of NEM1.

8.4 Approximation and Accuracy

No changes to either the text or the homework.

Area of Circles

Here is a circle with radius 10 cm. Estimate its area.

each square is 1 cm × 1 cm

15 cm²

7 × 7 cm²
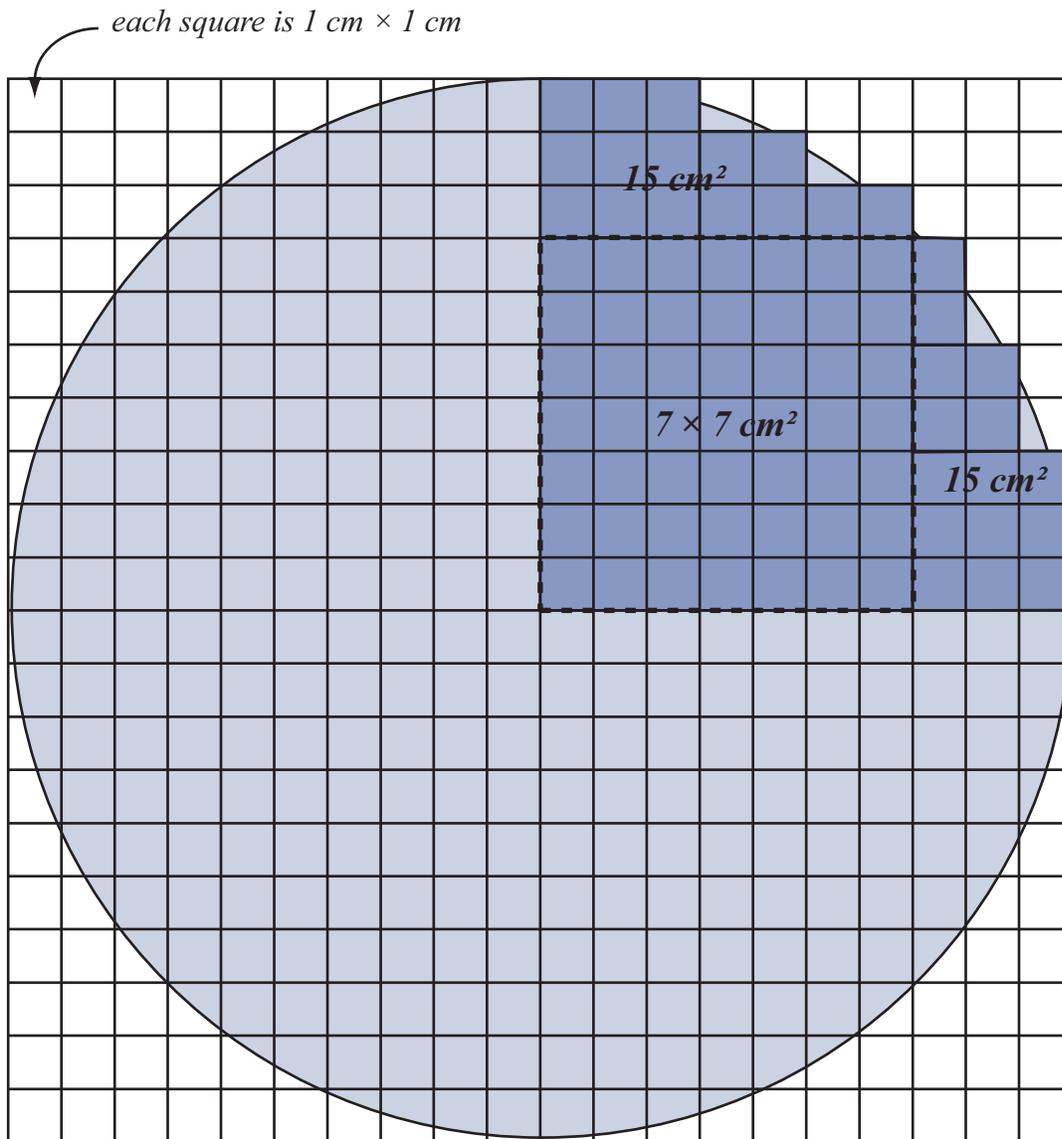
15 cm²

Area of a quarter-circle $\approx$ 49 + 15 + 15 cm²

$\qquad\qquad\qquad\qquad = $ 79 cm².

Area of the circle $\approx$ 4 × 79 cm²

$\qquad\qquad\qquad\quad = $ 316 cm².

Volume and Surface Area

9.1 Introducing Volume

Changes to text: Exercises 1.2 and 1.3 on page 195 refer to a set of marvelous "tank problems" in the U.S. edition of the Primary Math 5B. These problems do not appear in the Standards edition. As a replacement, a different set of problems appears on pages 31 – 35 below. The exercises on page 195 of the text can then be replaced by the following.

EXERCISE 1.2. *Study the grade 5 introduction to displaced volume shown on page 31 below. Notice that the beaker is marked in milliliters, but the answer is to be in cubic centimeters. In which grade did Primary Math students learn to convert between these?*

EXERCISE 1.3. *Carefully read and solve the first problem on page 32. Notice that this is a "Teacher's Solution".*

If you have access to the U.S. edition versions of Primary Math 5B and 6B, we recommend studying *all* of the tank problems in the order presented. These are engaging problems for grade 6 students, and they clearly show how the Primary Mathematics curriculum prepares students for middle school science.

Homework Set 32

Problems 1–5 refer to pages 137–146 of Primary Math 4B, where 4th grade students are introduced to volume.

1. (*Study the Textbook!*) Read pages 137–140 of Primary Math 4B. Page 140 shows the reasoning that gives the volume formula for cuboids. Write one sentence describing the point of the picture at the bottom of page 140.

2. Continue, reading pages 141–143 and writing the answers to all questions in your copy of the text. Then, for parts B, C, and D of Problem 6 on page 143, write clear answers that show how to do the multiplication mentally (remember, to multiply by 5, take half the number and multiply by 10).

3. (*Study the Textbook!*) The first problem on page 144 presents the definition of 1 liter and 1 milliliter, which is extremely important!

   a) Write down the three facts relating $1\,\ell, 1\,\mathrm{m}\ell$ and $1\ \mathrm{cm}^3$ that appear in this problem.

   b) Write answers for the remaining problems on page 144.

4. Do all of the problems on page 145 and write the answers as a list (write only the 7 answers, separated by commas).

5. Similarly list the answers to all problems on page 146.

6. (*Study the Textbook!*) Open Primary Math 5B and read Problems 8, 9 and 10 on page 52–53.

   a) Instead of giving the lengths of 3 edges and asking for the volume of a cuboid, what do these problems do?

   b) Which problem on these pages introduces the idea that the volume of a cuboid is (Area of base) × height?

7. Problem 11 on page 53 and Problems 18 and 20 on page 56 reinforce the idea that the volume of a cuboid is the area of the base times the height.

   a) Carefully read and answer these problems.

   b) What new idea is taught in Problem 11?

   c) The problems on this page introduce the idea of measuring volume using water, which is used in the next section. They also introduce the idea of converting between different units of volume — which ones?

   The next three problems refer to the grade 5 – 6 problems that appear on pages 31 – 35 below (in this booklet).

8. (*Study the Textbook!*) Page 31 introduces "Archimedes's principle": the idea that one can measure the volume of an irregular-shaped object by placing it in water and measuring how much water is displaced. Examine this page carefully (note the conversion between milliliters and cubic centimeters) and fill in the blanks.

9. Turn to the next page ("Finding displaced volume"). Read and solve Problems 1, 2, and 3. On your homework sheet, write down the numbers that go in the blanks in these problems. Notice how Problem 3 uses the principle that the volume of water does not change when it is moved to another container.

10. Continue, doing all 6 problems in Practice Set 1.

11. The four solids in the next column are formed from rectangular solids. Find the volume of each, making your method clear.

a)

b)

c)

d)
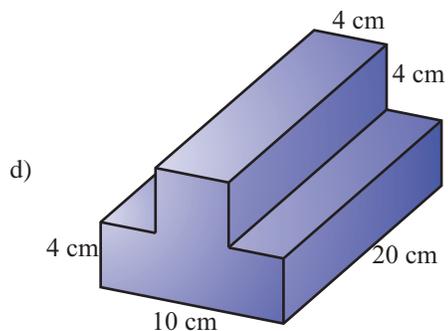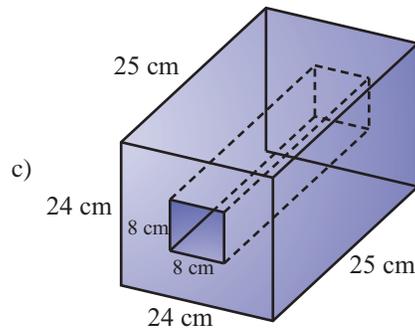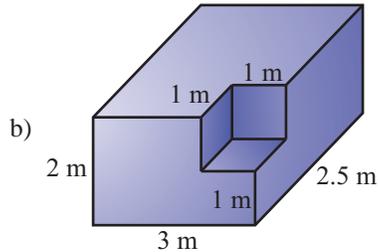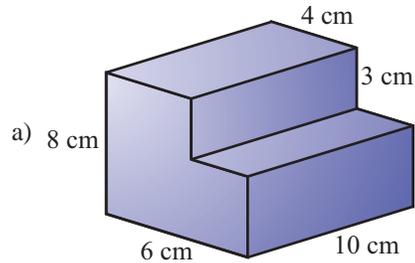
9.2 Metric Volume

Homework Set 33

1. Do the following area and volume conversions. Use the method of Examples 2.5 and 2.6 in this section:

   a) $2867 \text{ mm}^2 = $ _____ dm^2.

   b) $0.32 \text{ m}^3 = $ _____ cm^3.

   c) $58,300 \text{ mm}^3 = $ _____ m^3.

2. Convert by shifting decimal places:

   a) Length: $0.732 \text{ m} = $ _____ cm.

   b) Weight: $2867 \text{ g} = $ _____ kg.

c) Liquid volume: 7.4 ℓ = _____ mℓ.

3. When converting area units, the place value shift is twice what it is for the corresponding length units. Convert:

   a) 1200 cm = _____ m.
   b) 1200 cm^2 = _____ m^2.
   c) 2.6 cm^2 = _____ mm^2.
   d) 0.000237 km^2 = _____ m^2.

4. Similarly, for volume units, the place value shift is three times what it is for the corresponding length units. Convert:

   a) 25,000 cm = _____ m,
      25,000 cm^3 = _____ m^3.
   b) 8 dm^3 = _____ cm^3.
   c) 1032 mm^3 = _____ cm^3.

5. a) 12 dm^3 = _____ ℓ.
   b) 2300 cm^3 = _____ ℓ.
   c) 5 m^3 = _____ ℓ.

6. Density is introduced in Section 13.2 (pages 359–360) of NEM1. Read those pages and answer the following questions.

   a) Write down the definition of density.
   b) What is the density of water in grams per cubic centimeter? If something is more dense than water, than its density is (choose one): more than 1, less than 1.
   c) By substituting into the definition of density, answer problems 1a and 2a on page 360 of NEM1.
   d) Give a Teacher's Solution to Problem 5 on page 361 (follow the format of the example at the bottom of page 359).

e) Give a similar solution to Problem 6. Express the weight in kilograms.

7. Because units of time are not related by powers of 10, conversions involving time require more than shifting decimal points:

   a) 1 day = _____ sec.
   b) Speed: 10 $\frac{m}{sec}$ = _____ $\frac{km}{sec}$ = _____ $\frac{km}{hour}$.
   c) Flow rate: 9 $\frac{m\ell}{sec}$ = _____ $\frac{\ell}{sec}$ = _____ $\frac{\ell}{min}$.

8. Archimedes' Principle can be developed further through problems that involve rates. In these problems, water is poured into, or drained from, containers at so many liters per minute.

   a) Study and fill in the blanks in Problem 1 in Practice Set 2 on page 35 below (in this booklet).
   b) Give Teacher's Solutions (with pictures) to the remaining 3 problems in Practice Set 2. Aren't these fantastic problems?

9. ✳ (*Calculator*) The speed of light is approximately 300,000 km/sec.

   a) About how many miles per second is that? (1 mile = 1.6 km.)
   b) How many times around the earth can light travel in one second? Begin by drawing a sketch of the earth and noting that the distance from the equator to the north pole is 10 million meters.
   c) How many miles does light travel in a year (365.25 days)? This enormous unit of distance is called a light-year.

9.3 Prisms and Cylinders

Homework Set 34

1. The base of a right cylindrical tank is a circle with radius 70 cm. It contains 616 liters of water when it is $\frac{2}{3}$ full. Find the height of the tank in cm. (Take $\pi = \frac{22}{7}$.)

2. Fill in the chart at the right.

	base area	height	volume of prism
(a)	11 cm²	8 cm	
(b)	6 m²	21 m	
(c)		19 m	95 m³
(d)	16 cm²		64 cm³

In Problems 3-8, find the volume of the prism shown.

3.

3 cm
5 cm
10 cm

4.

Area: 4 m²
2.2 m

5.

Area: 12 cm²
5 cm

6.

5 m
4 m
8 m   10 m

7.

10 m
12 m
cylindrical hole
of diameter 6 m
20 m

8.

4 cm
3 cm
10 cm
2 cm   20 cm
12 cm
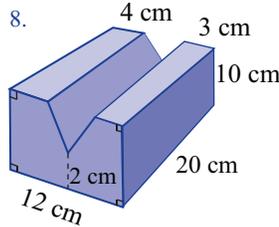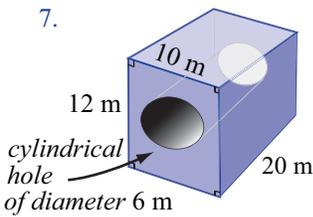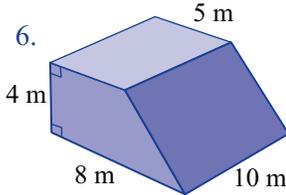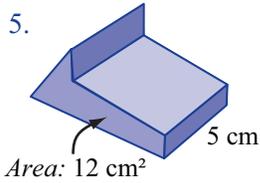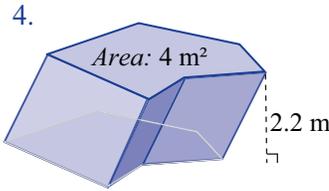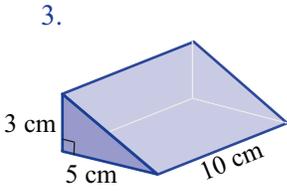
9. The cross-section of a steel beam (an "I-beam") is shown below. All angles are right angles, and the cross-section is symmetric both horizontally and vertically. If the beam is 3 m long,

   a) Find its volume in cm³.

   b) Find its volume in dm³.

   c) If it is made of steel with a density of 7.75 $g/$ cm³, find its weight in kilograms.

16 cm
5 cm
20 cm   6 cm
5 cm
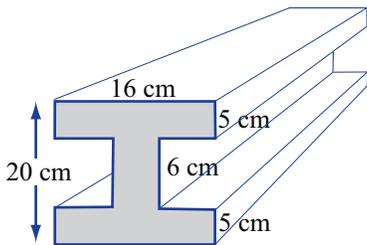
10. a) For a right prism, are all of the lateral edges the same length? Are they for an oblique prism?

   b) For a right prism, is the length of a lateral edge equal to the height? Is it for an oblique prism?

11. What is the total surface area of a cube whose volume is 27 cm³?

12. (*Calculator*) Taking $\pi = 3.14$, find the volume of

   a) A cylinder of radius 5 cm and height 20 cm.

   b) A cylinder of diameter 6 cm and height 6 cm.

13. (*Calculator*) A cylindrical water storage tank has a base diameter of 10 m and contains 219.8 m³ of water. Find the depth of the water. (Take $\pi = 3.14$.)

14. A cylinder with a base radius of 5 cm contains water with a depth of 9 cm. When a stone is immersed in the water, the depth of the water increases to 10.5 cm deep, completely covering the stone. What is the volume of the stone? Write your answer in terms of π.

5 cm
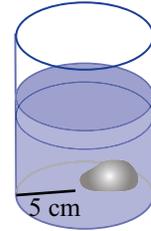
15. *Draw a picture and give a Teacher's Solution* to the following problem (in the last step, take $\pi = 3.14$ and use a calculator):

A solid metal cube 8 cm on each side is melted and recast into a solid cylinder with radius 10 cm. What is the height of the cylinder?

16. A solid circular cylinder has a circumference of 44 cm and a height of 10 cm. Find:

   a) The radius (Take $\pi = \frac{22}{7}$).

   b) The volume of the cylinder.

   c) The surface area of the curved surface of the cylinder.

   d) The total surface area (including top and bottom).

17. Do Problem 4 on page 364 of NEM1.

The final three problems illustrate some practical applications. Use a calculator for these.

18. Do Problem 4 on page 360 of NEM1.

19. ✱ Do Problem 19 on page 366 of NEM1.

20. ✱ One Calorie (1 Cal) is the amount of energy required to raise the temperature of 1 kg of water by 1° C.

   a) If you drink 500 $m\ell$ of cold water at 7° C, how many Calories do you use in heating it to body temperature (37° C)?

   b) A cylindrical pot 18 cm in diameter is filled with cold water to a depth of 8 cm and placed on a stove. If the initial temperature of the water is 12° C, how many Calories are required to heat it to 84° C? Use a calculator and take $\pi = 3.14$.

9.4 Pyramids and Cones

9.5 Spheres

No changes to either the text or the homework in Sections 9.4 and 9.5.

Volume of a solid

Maya poured 500 *ml* of water into a measuring beaker. When she added a rock, the water level rose to 700 *ml*. What was the volume of the rock?
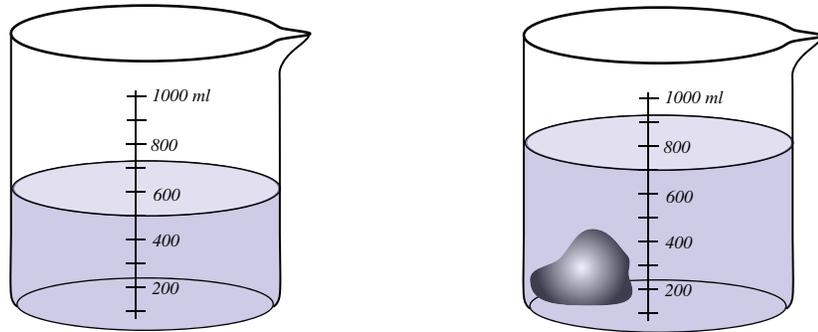
Volume of water: 500 cm³.

Volume of water and rock: 700 cm³.

Volume of rock: _____ cm³.

1 ml = 1 cm³

The rock *displaced* or moved _____ cm³ of water.

Volume of rock = Volume of water displaced

Finding displaced volume

1.   A rectangular fish tank, 30 cm long and 20 cm wide, contained water 5 cm deep. Then more water was added until the depth was 10 cm. How much water was added?
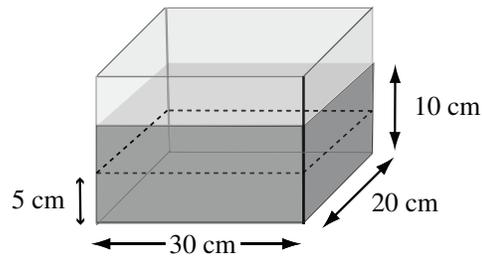
Increase in water height: 5 cm.

Volume of water added: $30 \text{ cm} \times \underbrace{20 \text{ cm} \times 5 \text{ cm}}_{100 \text{ cm}^2} = 3000 \text{ cm}^3$.

_____ liters of water were added.

1000 cm³ = 1 liter

2.   A rectangular tank is 50 cm long and 20 cm wide contained water 10 cm deep. When a rock was dropped in, the water level rose to 14 cm and completely covered the rock. What was the volume of the rock in cubic centimeters?
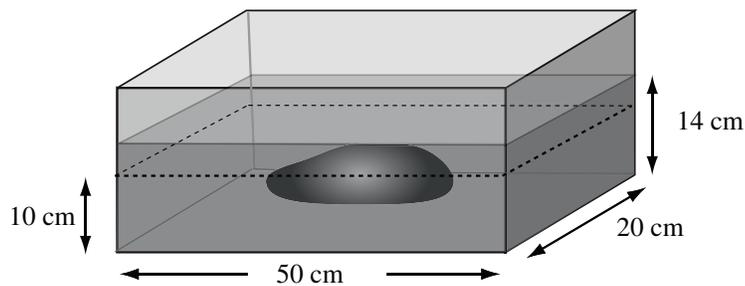
Increase in water height: 4 cm.

Volume of water added: $50 \text{ cm} \times 20 \text{ cm} \times 4 \text{ cm} = $ _____ cm^3.

The volume of the rock is _____ cm^3.

3.   Another rectangular tank, 40 cm long and 25 cm wide contains a rock and enough water to cover the rock. When the rock is removed, the water level drops 2 cm. What is the volume of the rock?
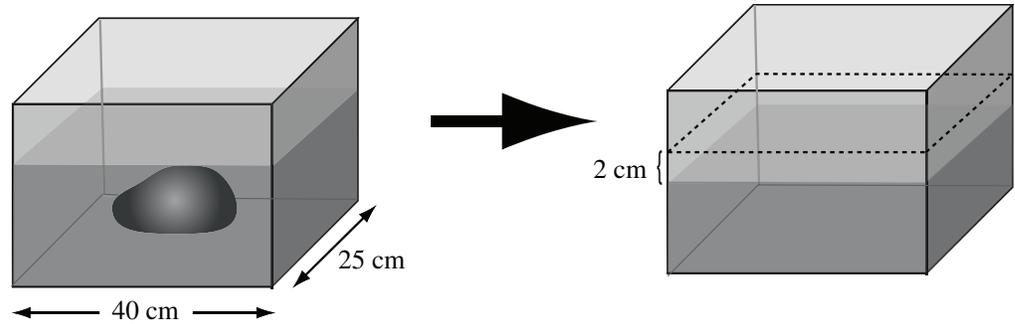

Drop in water height: 2 cm.

Volume of water displaced: 40 cm × 25 cm × _____ cm = _____ cm^3.

The volume of the rock is _____ cm^3.

Practice Set 1

1.   The base of a rectangular tank has an area of 900 cm^2. If the tank contains 3.6 liters of water, what is the height of the water level?


? cm

Area of base: 900 cm^2
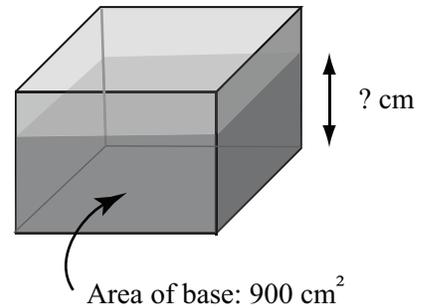
2. This box has volume 490 cm^3.
   Length of side TU = _____ cm.

7 cm

7 cm

T   ?   U
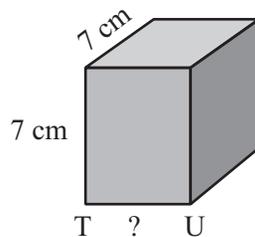
3. This box has volume 400 in³.
   Length of side TU = _____ in.


Area: 80 in^2

S

?

R
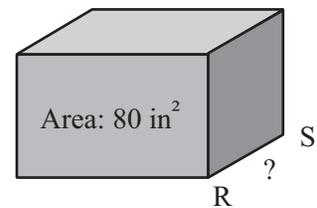
4.   Two rectangular containers have the dimensions shown. Container A is filled with water. All the water in Container A is then poured into Container B. The water filled Container B to what height?

10 cm

6 cm

6 cm

5 cm

12 cm

Container A   **Container B**
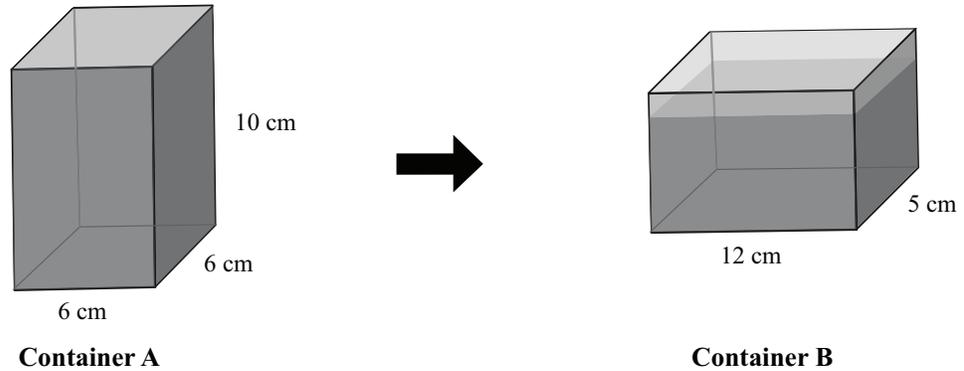
5.   A tank, 30 cm long and 20 cm wide, contained water 12 cm deep. A solid metal box with sides of length 10 cm, 5 cm and 6 cm was placed in the tank. What is the height of the water then?

6 cm

10 cm   5 cm

?

20 cm

30 cm
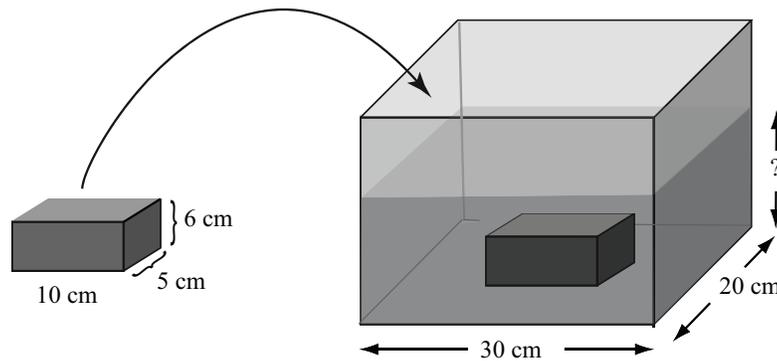
6.   Another rectangular tank, 10 cm long and 6 cm wide, contained water 4 cm deep. After some small stones were placed in the tank, the water level rose to 6 cm. What was the total volume of the stones?

6 cm

4 cm

6 cm

10 cm
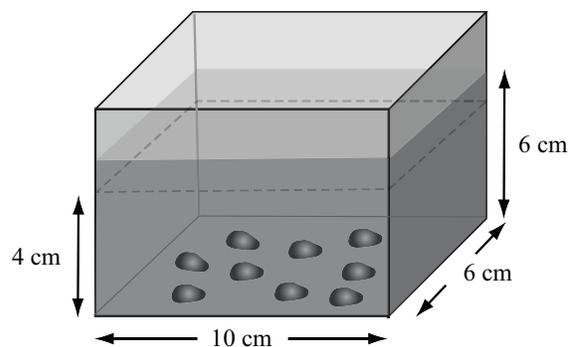
Practice Set 2

1.   An empty rectangular tank, 60 cm long and 50 cm wide and 30 cm tall, is filled with water at a rate of 4 liters per minute. How long will it take to fill the tank?
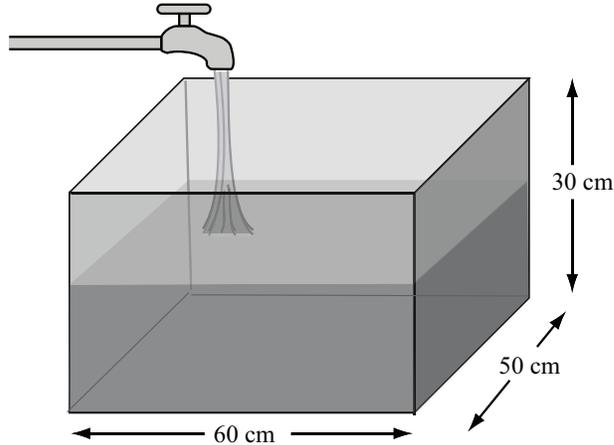
Volume of tank: 60 cm $\times$ 50 cm $\times$ 30 cm   = $(6 \times 5 \times 3) \times 1000 \text{ cm}^3$

=   _____ liters.

Time required to fill the tank: _____ $\div 4$ = _____ minutes.

2.   A rectangular tank, 40 cm long and 20 cm wide and 50 cm tall, is full of water. How long will it take to empty the tank if it is drained at a rate of 2 liters per minute?

3.   The base of a rectangular tank measures 50 cm long and 60 cm wide, and its height is 36 cm. It is $\frac{2}{3}$ full of water. How long will it take to fill the tank if water is poured in at a rate of 4 liters per minute?

4.   The base of a rectangular has an area of 600 cm². It has a height of 40 cm. Maria put a brick in the tank, and then poured in water from a faucet at a rate of 6 liters per minute. It took $3\frac{1}{2}$ minutes to completely fill the tank. What was the volume of the brick?

Data Displays, Probability and Statistics

10.1 Data Displays

Changes to text: The grade levels shown next to the figures on pages 224–226 refer to the U.S. edition of Primary Mathematics. In the Standards edition, the teaching sequence for graphs is accelerated to conform to the California state mathematics standards. The corresponding grade levels for the Standards edition are:

- Grade 1: Pictograms.

- Grades 2: Abstract pictograms with labels.

- Grades 2 & 3: Bar charts.

- Grades 4 & 5: Line graphs.

- Grades 6 − 8: as described on page 226.

Homework Set 37

1. (*Study the Textbook!*) In the Primary Math Standards Edition, pictograms are introduced in first grade and reviewed in second grade. Read pages 101–107 of Primary Math 2B, answering the questions mentally.

    a) How does the pictogram on page 103 differ from the one on page 102? What new concept is introduced?

    b) A teacher could emphasize the new concept by having students count the number of each fruit using skip-counting by what (twos, threes, fours or fives)?

    c) Questions (c)–(f) on page 103 are not as simple as those on page 102: they include comparisons ("how many more?") and what other type of question?

    d) When you present this example in front of the entire class, some of your students think the answer to (c) is 1. How could you modify the picture (adding detail) to reinforce the meaning of one triangle?

2. (*Study the Textbook!*) In the pictogram on page 103, each fruit has its own color.

    a) How would you respond to a student who asks "what kind of fruit do gray triangles stand for?"

b) Is any information lost if all triangles are made gray?

c) The textbook publisher added color to make the page look more appealing. In this case, does the color aid the teacher, or create distractions?

3. *(Study the textbook!)* Read pages 107–113 of Primary Math 2B and answer these questions:

a) Between which two pages does the transition from pictograms to bar graphs take place?

b) Write answers to the questions on page 109 in your textbook. How many different types of questions are asked here?

c) Answer questions (a)–(f) on page 110 in your textbook. Which of these ask students to read the chart "backwards", using dollar amounts to find a month?

d) Answer the questions on pages 113 in your textbook. Which of these require students to solve an arithmetic problem?

e) Assuming that 1 – 2 pages can be covered in a day's lesson, estimate how many class days would be spent on data analysis in second grade.

4. *(Study the textbook!)* When answering questions about bar charts, students can determine the height or length of bars by either

(i) counting squares

(ii) reading the scale, or

(iii) doing both.

In Primary Math 2B, which of these methods can students use to answer the question on page 104? The questions on page 108? Questions (a) on pages 109, 110, 113? What have students learned to do on pages 108 –110 and 113?

5. *(Study the textbook!)*   Read pages 140–143 of Primary Math 3A and write answers to the questions in your textbook. Then answer these questions:

a) Do problem 4 on page 143.

b) Do problem 5 on page 143.

c) Assuming that 1 – 2 pages can be covered in a day's lesson, estimate how many class days would be spent on data analysis in fourth grade.

6. Line graphs are introduced on pages 119–121 of Primary Math 4B. Imagine that you are teaching these pages to a fifth grade class. Write down a 3-part question about the graph on page 121 that would help prepare your students to learn about the slope of lines.

7. Do Problems 1–6 on pages 131–134 of Primary Math 5B.

8. *(Study the textbook!)* Now reread pages 131–134 of Primary Math 5B and answer these questions:

a) List the questions in Problems 1–6 that require students to calculate a fraction (your answer should start with "1c, 2a, ...")

b) List the questions in Problems 1–6 that require students to calculate a percent.

c) List the questions in Problems 1–6 that require students to do arithmetic.

d) Give a Teacher's Solution for Problem 2c).

e) For Problem 6b) on page 134, what geometric fact is used in determining the percentage of shirts? (See the list of facts at the beginning of Chapter 4 in this textbook.)

10.2 Center and Dispersion of Data Sets

Changes to text: Page 229, line -2 should read "pages 106–108".

Homework Set 38

1. *(Study the textbook!)* In Primary Math 5B, read page 106 and do Problems 1 and 2 on page 107.

a) Is the picture in Problem 1 essentially the same as the area model at the beginning of this section?

b) Give a short explanation on how to use the pictogram in Problem 2 to *quickly* find the average.

2. Do parts (a), (f), and (g) of Problem 1 on page 110 of Primary Math 5B.

3. Find the mean, median, and mode of the following sets below:

a) 1, 2, 2, 3, 4, 5.

b) -20, 2, 3, 2, -1, 2, -1, 4, -1.

c) 15 cm, 3 dm, 1 dm, 20 cm, 1 dm.

    d) 4 quarts, 3 pints, 6 cups, 2 quarts, 1 gallon, 4 pints.

4. Give Teacher's Solutions to Problems 2–8 on page 110 of Primary Math 5B.

5. Do Problem 2 on page 120 of Primary Mathematics 4B. Referring to the graph, find the average weekly sales.

6. Mr. Smith's daily expenses for the 7 days in one week were $120, $72, $58, $233, $82, $91 and $108. What was his mean daily expense? What was his median of his daily expenses?

7. Give a Teacher's Solution to this problem: The mean of 5 numbers is 12 and the mean of 9 other numbers is 22.8. What is the mean of all 14 numbers?

8. Give a Teacher's Solution to this problem: Katie's mean score on 4 hourly exams was 88%. Course grades are computed from the 4 hourly exams and the Final Exam, which counts as 2 hourly exams. What score must Katie get on the Final Exam to achieve a course grade of 90% ?

9. Imagine that you are teaching from a textbook that gives this algorithm for finding the first quartile: Q_1 *is the median of the data points that are less than, but not equal to, the median* Q_2.

    a) Use this algorithm to find Q_1 for the following two data sets:

      Group A: 102, 107, 110, 111, 112, 112, 123, 136.

      Group B: 98, 100, 102, 105, 112, 113, 114, 119, 128.

    b) Find Q_1 for both groups again, now using the algorithm you described in Problem 9.

    c) For Group B, are your two values for Q_1 the same?

10. Another textbook says: Q_1 *is the median of the lower half of the data (do not include the median in the lower half).* Two students disagree about how this applies to the data:

    Group C: 50, 62, 70, 73, 77, 77, 80, 81, 85, 90, 94.

    a) Annie drops both 77s and gets $Q_1 =$ _____.

    b) Jorge drops only one 77 and gets $Q_1 =$ _____.

    c) Is the textbook's definition ambiguous?

    d) How might you, as a teacher, resolve it?

11. Draw box plots for the Group A and Group B data given in Problem 10 above.

12. Six children ran a 50 meter dash. Their times were 9, 10, 10, 11, 12 and 14 seconds. Find the mean deviation MD, the variance Var and the standard deviation σ. Express your answers in the appropriate units.

10.3 Probability

10.4 Inferential Statistics

No changes to the text or the homework in Sections 10.3 and 10.4.