CHAPTER 3

Finding Unknown Angles

Geometry becomes more interesting when students start using geometric facts to find unknown lengths and angles. During this stage, roughly grades 5-8, students work on "unknown angle problems". These problems are learning bonanzas. They initiate students in the art of deductive reasoning, solidify their understanding of geometry and measurement, and help introduce algebra.

You have already solved some unknown angle problems and seen how they are integrated into the Primary Math curriculum in grades 5 and 6. This chapter examines how unknown angle problems are used to develop geometry in grades 6 and 7.

From a teaching perspective, unknown angle problems are not just part of the geometry curriculum, they *are* the curriculum in grades 5-8; everything else is secondary. In these grades, teachers and textbooks introduce facts about angles within triangles and polygons, about parallel lines, about congruent and similar figures, and about circles. These are not simply facts to memorize: understanding emerges as students use them to solve problems. Thus teaching centers on *solving problems*.

Unknown angle problems are superbly suited for this purpose. Solutions require several steps, each applying a known fact to the given figure. As students do these problems the geometric facts spring to life; these facts become friends that can be called upon to help solve problems. Unknown angle problems are also enormous fun!

3.1 Unknown Angle Problems

An unknown angle problem is a puzzle consisting of a figure with the measures of some sides and angles given and with one angle — the unknown angle — marked with a letter. The student's task is to find the measure of the unknown angle by applying basic geometric facts. Beginning exercises require only rudimentary facts, such as the fact that angles around a point add to 360°. As new geometric facts are introduced, they are added to the list of facts that are available as tools to solve unknown angle problems. As more knowledge is integrated, the problems become more challenging and more interesting.

This section examines the role of unknown angle problems in the Primary Math and New Elementary Math textbooks for grades 5-7. It includes a list of the geometric facts learned during this stage and a format for presenting "Teacher's Solutions" to unknown angle problems. You will be asked to use this format for many homework problems.

Many elementary textbooks, including the Primary Math books, introduce new concepts using the following specific process.

Teaching sequence for introducing geometric facts

1. Review background knowledge and introduce any new terms needed.
2. Introduce the fact by an activity (measuring, folding, or cutting-and-rearranging) that serves to *clarify what the fact says* and *convince students that it is true*.
3. Summarize by stating the geometric fact in simple clear language.
4. Have students solve dozens of unknown angle problems:
   a) simple problems using the fact alone,
   b) multi-step problems using the fact alone,
   c) multi-step problems combining the fact with previously-learned facts.

Step 3 takes only a few minutes, but it is the teacher's most important input. In geometry, words have precise meanings; students' success depends on knowing definitions and knowing how to apply them. One can even argue that geometry is included in the K-12 curriculum to teach students that *giving words precise meaning fosters clear thinking*. This lesson is applicable to all subjects.

After these preliminaries, the fun begins as students solve increasingly challenging problems (Step 4). As always in mathematics, the real learning occurs as students solve problems.

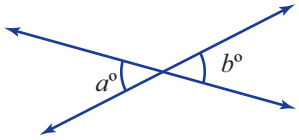
Geometry Facts — First List

As you have seen in homework problems, the basic facts about angles, triangles and quadrilaterals are presented in Primary Mathematics 5A and 5B. Below is a list of the facts learned at that stage. Each has a simple abbreviation. You will be expected to be consistent in using these abbreviations in your homework solutions.

The list of facts is built around three exercises. These questions ask you to observe how these facts are justified at the grade 5 level (using folding, cutting, and measuring exercises) and to observe the type of problems students are asked to solve.

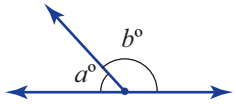
EXERCISE 1.1 (Angle Facts). *The following three facts are introduced on pages 85–88 of Primary Mathematics 5A. How are these facts justified?*

Vertical angles have equal measure.
(Abbreviation: **vert.** ∠s.)



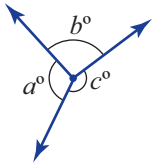
$a = b.$

The sum of adjacent angles on a straight line is 180°. (Abbreviation: ∠**s on a line**.)



$a+b = 180.$

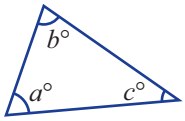
The sum of adjacent angles around a point is 360°.
(Abbreviation: ∠**s at a pt.**)



$a+b+c = 360.$

EXERCISE 1.2 (Triangle Facts). *The following five triangle facts are introduced on pages 57–64 in Primary Mathematics 5B. Locate the statement of each in your 5B book. What activity is used to justify the first fact? What wording is used for the fourth one?*

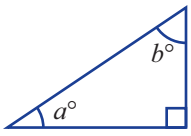
The angle sum of any triangle is 180°.
(Abbreviation: ∠ **sum of** Δ)



$a + b + c = 180.$

When one angle of a triangle is a right angle, the other two angles add up to 90°.

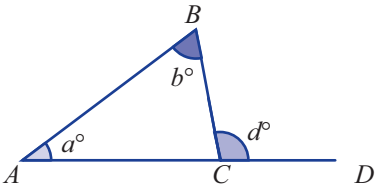
(Abbreviation: ∠ **sum of rt.** Δ.)



$a + b = 90.$

The exterior angle of a triangle is equal to the sum of the interior opposite angles.

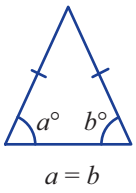
(Abbreviation: **ext.** ∠ **of** Δ.)



$d = a + b.$

Base angles of an isosceles triangle are equal.

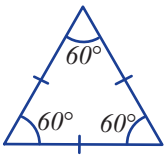
(Abbreviation: **base ∠s of isos.** Δ.)



$a = b$

Each interior angle of an equilateral triangle is 60°.

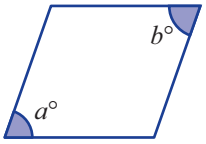
(Abbreviation: **equilat.** Δ.)



EXERCISE 1.3 (Quadrilateral Facts). *The next section of Primary Math 5B (pages 68–71) introduces two facts about 4-sided figures. Study the folding and cutting exercises given on page 70. How would you use these exercises in your class?*

Opposite angles in a parallelogram are equal.

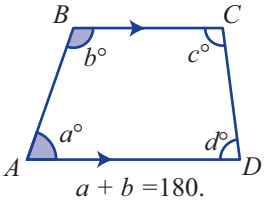
(Abbreviation: **opp. ∠s //-ogram**.)



$a = b.$

Interior angles between two parallel sides in a trapezoid (or a parallelogram) are supplementary.

(Abbreviation: **int. ∠s,** $\overline{BC} \,//\, \overline{AD}$.)



$a + b = 180.$
$c + d = 180.$

The "Teacher's Solution" Format for Unknown Angle Problems

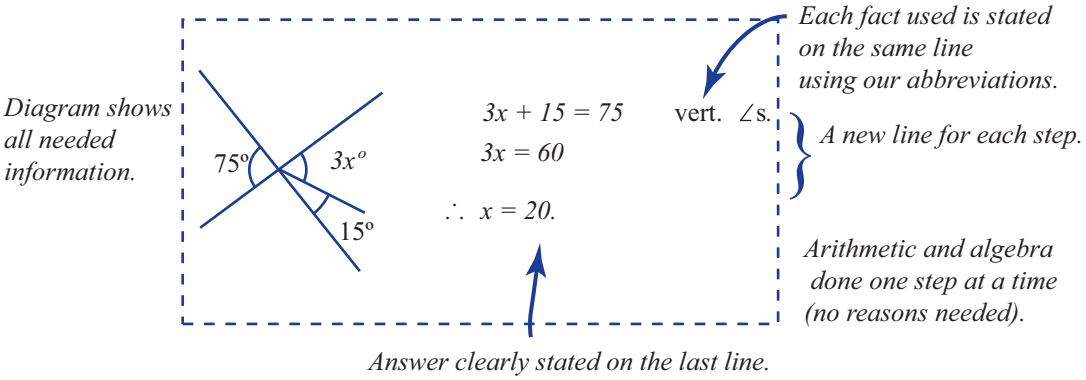
Teachers are obliged to present detailed solutions to problems for the benefit of their students. The teacher's solutions must meet a different standard than the students' solutions. Both teachers and students are expected to get the reasoning and the answer correct. But teacher-presented solutions must also communicate the thought process as clearly as possible.

In this book, solutions that meet this high standard are called **Teacher's Solutions**. You will frequently be asked to write such Teacher's Solutions in homework. If you are unsure how to do this, *look in the textbooks*: almost every solution presented in the Primary Math books, and all of the "Worked Examples" in the New Elementary Mathematics book, are Teacher's Solutions.

You are already familiar with one type of Teacher's Solution — bar diagrams. Bar diagrams are extraordinarily useful for communicating ideas about arithmetic. Teachers need similar devices for communicating geometric ideas. As a start, in this chapter you will be writing Teacher's Solutions for unknown angle problems. Here is a simple example.

EXAMPLE 1.4. *The figure shows angles around a point. Find the value of x.*

Teacher's Solution to an Unknown Angle Problem

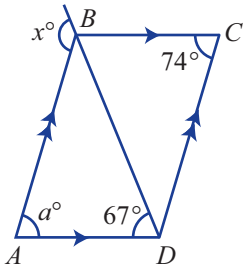


This solution is short and clear, yet displays all the reasoning. It always begins with a picture showing all points, lines, and angles used in the solution, and it always ends with a clear answer to the question asked.

Notice what happens with the degree signs. The angles in the picture have degree signs, so 75, 15 and $3x$ are all numbers. Thus we can drop the degree signs in the equations. This saves work and makes the solution clearer. Degree signs are handled in the same way in the next two examples.

EXAMPLE 1.5. *The figure shows a parallelogram. Find the value of x.*

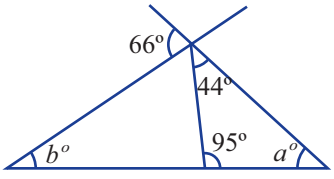
Teacher's Solution:



$a = 74$ opp. ∠s of //-ogram
$x = a + 67$ ext. ∠ of ΔABD
$\therefore x = 74 + 67$
$= 141.$

EXAMPLE 1.6. *Find the values of a and b in the following figure.*

Teacher's Solution:



$a + 44 + 95 = 180$ ∠ sum of Δ
$a + 139 = 180,$
$\therefore a = 41.$

$a + b = 66$ ext. ∠ of Δ
$41 + b = 66,$
$\therefore b = 25.$

Elementary students are not usually asked to record reasons in the manner done in these examples. But in the New Elementary Mathematics textbook, middle school students are expected to present solutions exactly as above — middle school students use the same format as elementary school teachers!

This Teacher's Solution format is used by

- *Teachers* in elementary school,
- *Teachers and students* in middle school.

Unknown angle problems in grades 5-7 also help introduce algebra. These geometry problems expose students to the idea of using letters to stand for numbers (notice that the whole alphabet, not just x and y, are used!). They often require solving simple linear equations, as you see in Example 1.4. This is a sneaky trick: giving students practice in algebra as they do geometry. Solving for unknown angles also gives students a visual way to understand what it means to "solve for x" and appreciate why one would want to.

Homework Set 10

1. (*Study the Textbook!*) Give one-sentence answers to Exercises 1.1, 1.2, and 1.3 in this section.
2. (*Study the Textbook!*) Turn to page 87 of Primary Math 5A.
   - Do all of the problems on pages 87 and 88. Write your answers as a single list of numbers, separated by commas, but *not* labeled by the problem numbers.
   - Note the variation: on pages 87 and 88, how many different letters are used to stand for numbers?
3. On page 109 of Primary Math 6B, answer Problems 35, 36, and 37.
4. Answer Problem 2 on page 264 of NEM1. Show your reasoning.
5. For Problem 3 on page 264 of NEM 1, write Teacher's Solutions to parts b, d, e, g, and j.
6. For Problem 4 on the next page (page 265) write Teacher's Solutions to parts b, c, f and g.
7. (*Study the Textbook!*) Compare the grade 6 and grade 7 problems you did in this HW Set. Name two new features required in solving the grade 7 problems that are not present in the grade 6 problems.